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Specialization and wages when jobs generate utility

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Abstract

The paper presents a model where both income and hours of work are allowed to have a positive effect on individual utility. Increasing returns to specialization and the concavity of the utility function leads some workers to choose lower wages in return for a more interesting job. Such interest in ones' work means that not all possible gains from specialization are achieved and that the correlation between income and talent (opportunities) is weakened. If letting workers do many tasks is costly for the firms, there may be few jobs where the workers can choose to be a generalist.

Keywords: labor supply, skills, wages, increasing returns

JEL codes: J24, J31, J32

1 Introduction

While the general aspects of wage determination are well understood by economists, there are many details which can seem puzzling. This is of course to be expected, given the wide variety of individuals and jobs that exist, and the differing characteristics of different labor markets. Even so, some puzzling details may reveal deeper structural aspects of wage determination. In the following I will look more closely at why wages in equilibrium can vary between different jobs offered to the same individual. The explanation I propose will probably be familiar to academics who sometimes feel little appreciated and lowly paid. The cost of having an interesting job is to have lower wages than one could otherwise get.

In the table below, monthly labor income for full time employees is given for some different position in Norway. Wages vary strongly, being highest in industry and lowest in government. There are of course huge amounts of unexplained heterogeneity behind these figures (there can among other things be differences in age and type of education.), but they probably truthfully indicate that an individual will receive higher wages working in the private sector than in the public

sector. This is a situation which has lasted for many years, so it seems to reflect some type of equilibrium. One might hypothesize that jobs giving lower income compensate their employees by giving them work which they find personally interesting or important. There are many attempts to quantify the degree of wage dispersion, see for example Blackburn and Neumark (1992) and Gibbons and Katz (1992), generally finding large inter-industry wage differentials for workers with identical characteristics. There is a large literature on wage dispersions

Table 1. Monthly compensation in Norwegian kroner for full time employees, 2001

	Labor Income
Industry	
- <i>management</i>	40 889
- <i>academic position</i>	32 751
Central Government	
- <i>research position (forsker)</i>	30 346
Public Schools	
- <i>working at a high school</i>	27 867

Statistisk Årbok 2002, Statistics Norway, Oslo.

The model presented in the following assumes that being employed can bring direct utility in addition to the income the job generates. This is combined with considering the effect on specialization within the work place. We assume that in each type of work there are different tasks and that there are increasing returns to each task. If work only brings disutility, one would expect workers to specialize at only one task. In our model this is not necessarily the case. The effect of being interested in ones profession can be thought of as implying a desire to keep abreast of all developments at the expense of specialization. Individuals with a high interest in their profession will then have a higher likelihood of being generalists, and not capture all the possible gains from specialization. We thereby get a second, income lowering effect. Some individuals have lower wages than the maximum they could achieve in the market because they receive more personal satisfaction from their job than other jobs can give and because their interest in their jobs leads them to forego some returns to specialization.

I will distinguish between utility generating jobs and non-utility generating jobs. One might want to associate the first type with knowledge based jobs. This is a simplification, all the jobs listed in the table above could probably be described as knowledge based. Even so, this association can be valuable in thinking about issues concerning research and development and economic growth (though this will not be further discussed in the following). One might for example want to consider

what split between generalists (with overview) and specialists (expanding the frontier) is optimal from a knowledge accumulation stand point.

The contrast between increasing returns to specialization and a desire to work on many different issues is central to the paper. This trade-off has not been much discussed in the labor market literature, but there has been done much work on specialization and the labor market. Cheng and Yang (2004) give an excellent survey of the literature on inframarginal analysis of division of labor. Inframarginal decisions concern what activities to engage in, in contrast to marginal decisions which determine how resources are allocated to already chosen activities. Much of this literature has its roots in Adam Smith's (Smith, 1776) argument that the size of markets put limits on the degree to which labor can specialize, and Allyn Young's (Young, 1928) opposite assertion that the extent of markets depends on the degree to which labor is specialized. Many modern models such as Diamantaras and Gilles (2004) are based on the notion of consumer-producers, with Rader (1964) considering general equilibrium and Diamantaras and Gilles (2004) the individuals' maximization problem under transaction costs. In the following I consider the firm to have standard non-increasing returns properties, but that within a job workers can experience increasing returns to specialization. In other words the increasing returns I discuss are only considered at the level of the worker and does not consider the wider discussion of the specialization of labor.

An approach that is closely related to my paper is the paper by Mani and Mullin (2004), which discusses how social perceptions about occupations can lead to misallocation. In an overlapping generations model they model workers who choose jobs so as to maximize the utility of wages and social approval. Assuming that talent is easier to spot in popular jobs, they get an over-representation of high talent individuals in the popular job (where their talent will be seen and appreciated). Another related approach is Bhaskar and To (2003) who model wage dispersion in an imperfectly competitive labor market. As in my approach, they assume that worker with identical skills have heterogenous preferences over non-wage characteristics of jobs, but in contrast assume that the marginal product of labor varies between employers and find the general equilibrium wage distribution.

2 Two categories of work and two tasks

Work is assumed to fall into one of two categories: utility generating work and non-utility generating work. Each type of work consists of many different tasks, where workers can specialize by concentrating on a few or only one task. We assume that efficiency increases when a worker specializes.

Each individual chooses which type of work to do and to what degree she or he wishes to

specialize within the chosen type of work. The choice is determined by the individuals' talents and preferences for working on many different tasks.

As a starting point, we look at a very simple model with two types of work each consisting of two different tasks. One type of work is utility generating, while the other is not. Individual i is endowed with a talent vector $(a_{i1}, a_{i2}, b_{i1}, b_{i2})$, where a_{i1} and a_{i2} are the individuals' talent for doing tasks 1 and 2 in the professional line of work, while b_{i1} and b_{i2} are the talent for doing tasks 1 and 2 in the non-professional line of work.

An illustration could be of a labor market for academics where working in government doing research (a_{i1}) and teaching (a_{i2}) gives utility, while management (b_{i1}) and analysis in the private sector does not. This black white description is of course an exaggeration and is only meant as an illustration consistent with the initial table of wages. Many would probably argue that working in the private sector is more interesting than working in government.

2.1 Production

Production in the utility generating sector by individual i working can consist of up to two tasks, one yielding Y_{i1} and the other yielding Y_{i2} . Working l_{i1}^Y hours at the first task and l_{i2}^Y hours at the second, the individuals' production is given by the simple exponential functions

$$Y_{i1} = a_{i1} (l_{i1}^Y)^\alpha \tag{1}$$

and

$$Y_{i2} = a_{i2} (l_{i2}^Y)^\alpha, \tag{2}$$

where $a_{ik} > 0$ are the talent parameters defined above and α is a productivity parameter that is equal between tasks.

Goods production by individual i in a non-professional capacity also can consist of up to two tasks, one yielding Z_{i1} and the other yielding Z_{i2} . Letting hours worked at these tasks be given by l_{i1}^Z and l_{i2}^Z , production is assumed to be determined by

$$Z_{i1} = b_{i1} (l_{i1}^Z)^\beta \tag{3}$$

and

$$Z_{i2} = b_{i2} (l_{i2}^Z)^\beta, \tag{4}$$

with β being a productivity parameter.

We assume that the productivity parameters α and β are greater than 1, so that there are increasing returns to specializing in one task over the other within a type of work.

Total output is given by X

$$X = f \left(\sum_i Y_{i1}, \sum_i Y_{i2}, \sum_i Z_{i1}, \sum_i Z_{i2} \right) \quad (5)$$

We assume that wages are determined by the marginal productivity of labor, with the piece rate wage per unit of personal output Y_{ik} or Z_{ik} being denoted respectively as w_{Yk} and w_{Zk} . In the following, we do not discuss the general form of the production function f but take a set of equilibrium wages as given, assuming that they are such that the required number of workers of different types are recruited.

2.2 Utility

Individuals are assumed to derive utility from income and from work in the production of Y_{i1} and Y_{i2} . Work done in the in the production of Z_{i1} and Z_{i2} does not yield utility. Individual i 's utility function u_i has the form

$$u_i(c_i, l_{i1}^Y, l_{i2}^Y) = \begin{cases} \gamma_i \ln c_i + \delta_i \ln l_{i1}^Y + \delta_i \ln l_{i2}^Y & \text{if } l_{i1}^Y > 0, l_{i2}^Y > 0 \\ \gamma_i \ln c_i + \delta_i \ln l_{i1}^Y & \text{if } l_{i1}^Y > 0, l_{i2}^Y = 0 \\ \gamma_i \ln c_i + \delta_i \ln l_{i2}^Y & \text{if } l_{i1}^Y = 0, l_{i2}^Y > 0 \\ \gamma_i \ln c_i & \text{if } l_{i1}^Y = 0, l_{i2}^Y = 0 \end{cases} \quad (6)$$

where the parameter δ_i indicates the utility derived from hours worked at production types 1 and 2, while the parameter γ indicates the utility derived from consumption, c_i . Increasing returns will lead to corner solutions, so it is important to define the utility function at zero working hours. The utility of working in the knowledge sector, as captured by parameter δ_i , is assumed to vary between individuals.

Individuals who are very interested in their chosen sector of work have high δ -s and wish to work at many task. If they are not very interested, the δ -s are low and they are mainly concerned with their income and thereby consumption levels.

Consumption c_i equals income

$$c_i = w_{Y1}Y_{i1} + w_{Y2}Y_{i2} + w_{Z1}Z_{i1} + w_{Z2}Z_{i2} \quad (7)$$

while total hours worked, h , is assumed to be exogenously given.

3 Utility maximization

In principle, individuals can choose to work in both sectors doing all four possible tasks, but the increasing returns to specialization make this unlikely. We therefore rephrase the problem from

an optimization problem in the four variables l_{i1}^Y , l_{i2}^Y , l_{i1}^Z and l_{i2}^Z to a problem in the four derived variables L_{Yi} , L_{Zi} , h_i^Y and h_i^Z defined as follows:

$$\begin{aligned} h_i^Y &= l_{i1}^Y + l_{i2}^Y & L_{Yi} &= \frac{l_{i1}^Y}{h_i^Y} \\ h_i^Z &= l_{i1}^Z + l_{i2}^Z & L_{Zi} &= \frac{l_{i1}^Z}{h_i^Z}. \end{aligned} \quad (8)$$

When, in addition, we take into account that total hours must be equal to the exogenous constraint

$$h = h_i^Y + h_i^Z, \quad (9)$$

the number of endogenous variables is reduced to the three variables L_{Yi} , L_{Zi} and h_i^Y .

To simplify the notation we also introduce the variables

$$\left. \begin{aligned} A_{Yki} &= w_{Yk} a_{ik} \\ A_{Zki} &= w_{Zk} b_{k1} \end{aligned} \right\} \text{ for } k = 1, 2, \quad (10)$$

which can be viewed as talent adjusted piece wage rates.

Reformulating the problem using these variables and inserting the consumption equation (7) and the four task functions (1) - (4) into the utility function (6) leads to

$$\begin{aligned} U(L_{Yi}, L_{Zi}; h_i^Y > 0) &= \gamma_i \ln \left(A_{Y1i} (h_i^Y)^\alpha (L_{Yi})^\alpha + A_{Y2i} (h_i^Y)^\alpha (1 - L_{Yi})^\alpha \right. \\ &\quad \left. + A_{Z1i} (h - h_i^Y)^\beta (L_{Zi})^\beta + A_{Z2i} (h - h_i^Y)^\beta (1 - L_{Zi})^\beta \right) \\ &\quad + \delta_i \ln L_{Yi} + \delta_i \ln (1 - L_{Yi}) + 2\delta_i \ln h_i^Y, \quad (11) \end{aligned}$$

when $h_i^Y > 0$ and

$$\begin{aligned} U(L_{Yi}, L_{Zi}; h_i^Y = 0) &= \gamma_i \ln \left(A_{Y1i} (h_i^Y)^\alpha (L_{Yi})^\alpha + A_{Y2i} (h_i^Y)^\alpha (1 - L_{Yi})^\alpha \right. \\ &\quad \left. + A_{Z1i} (h - h_i^Y)^\beta (L_{Zi})^\beta + A_{Z2i} (h - h_i^Y)^\beta (1 - L_{Zi})^\beta \right), \quad (12) \end{aligned}$$

when $h_i^Y = 0$.

3.1 Specialization within a type of work

The concavity of the utility function implies that individuals will always want to work a little in the knowledge sector, but most individuals will be close to choosing only one type of work. To simplify the analysis, we assume that individuals only work in one sector. Specifically we assume that there is a minimum level of h_i^Y , denoted h_{\min} , that is such that at all higher levels individuals will choose either to only work in the utility generating sector, $h_i^Y = h$, or in the non-utility generating sector, $h_i^Z = h$. We denote the two corresponding utility functions as $U_Y(L_{Yi}, h_i^Y = h)$ and $U_Z(L_{Zi}, h_i^Y = 0)$. In the case of $h_i^Y = h$ the budget constraint will be

$$c_i = w_{Y1} Y_{i1} + w_{Y2} Y_{i2} \quad (13)$$

and in the case of $h_i^Y = 0$ it will be

$$c_i = w_{Z1}Z_{i1} + w_{Z2}Z_{i2}. \quad (14)$$

Maximizing the utility function

$$U_Y(L_{Yi}, h) = \gamma_i \ln(A_{Y1i}(h)^\alpha (L_{Yi})^\alpha + A_{Y2i}(h)^\alpha (1 - L_{Yi})^\alpha) + \delta_i \ln L_{Yi} + \delta_i \ln(1 - L_{Yi}) + 2\delta_i \ln h \quad (15)$$

wrt. L_{Yi} leads to the first order derivatives which can be either maximum or minimum points. In such a situation it is important to consider second order derivatives. Not surprisingly, we get the result that individuals who work in the utility generating sector specialize less than those in the other sector as stated in proposition 1.

Proposition 1 *Workers wishing to work in the utility generating sector ($h_i^Y = h$ and $h_i^Z = 0$) will choose to work at both tasks, but working longer hours at the task with the highest talent adjusted wage A_{Yki} . The first order condition*

$$\frac{\delta}{\gamma_i \alpha} - \left(\frac{2\delta_i}{\gamma_i \alpha} + 1 \right) L_{Yi} + \frac{w_{Y1}Y_{i1}}{w_{Y1}Y_{i1} + w_{Y2}Y_{i2}} = 0 \quad (16)$$

applies. Workers wishing to work in the non-utility sector ($h_i^Y = 0$ and $h_i^Z = h$) will choose a specialization strategy, only working at the task with the highest talent adjusted wage A_{Zki} .

Proof. See appendix A. ■

The first order condition is an implicit equation in L_{Yi} which we can not explicitly solve, except in special circumstances. Even so, it is fairly simple to analyze by decomposing it into the two functions

$$V_1(L_{Yi}) = \left(\frac{2\delta_i}{\gamma_i \alpha} + 1 \right) L_{Yi} - \frac{\delta_i}{\gamma_i \alpha} \quad (17)$$

$$V_2(L_{Yi}) = \frac{w_{Y1}Y_{i1}}{w_{Y1}Y_{i1} + w_{Y2}Y_{i2}} \quad (18)$$

and analyzing where they cross. For more details, see the appendix. It should be noted that when $A_{Y1i} = A_{Y2i}$, there are two solutions. In such a case one must check to see which gives the maximum utility.

Individuals working in the non-utility generating sector are only faced with increasing returns and therefore specialize fully. The trade-off between utility of consumption and work in the utility-generating sector arises from balancing the desire to work at all tasks with the increased income achieved by only working in at one task. Individuals who prefer to work in many fields will specialize less than those with lower preference for working in many fields. The larger δ_i an individual has, the more of a generalist she or he will be, but with a reduced income.

As economists, we have probably reflected on the conflict between keeping abreast of developments in the many different areas our subject covers and the need to work within our own speciality. Many became economists because they had a general interest in all matters economic, but find that the best career choice is to specialize in one or two areas.

3.2 Ensuring only one type of work (sector) is chosen

As assumed above there is a minimum constraint h_{\min} on working in the knowledge sector, leading all individuals to work in only one sector. In the following we give this condition a mathematical form (all possible interior solution points are minimum points with positive second order derivatives). The first order derivative, when $h_i^Y > 0$, is given by

$$\frac{\partial U(L_{Yi}, L_{Zi}, h_i^Y)}{\partial h_i^Y} = \frac{1}{c_i} \left\{ \alpha \gamma_i C_1 (h_i^Y)^{\alpha-1} - \beta \gamma_i C_2 (h - h_i^Y)^{\beta-1} \right\} + 2\delta_i (h_i^Y)^{-1}. \quad (19)$$

where

$$\begin{aligned} C_1 &= A_{Y1i} (L_{Yi})^\alpha + A_{Y2i} (1 - L_{Yi})^\alpha = (w_{Y1} Y_{i1} + w_{Y2} Y_{i2}) (h_i^Y)^{-\alpha} \\ C_2 &= A_{Z1i} (L_{Zi})^\beta + A_{Z2i} (1 - L_{Zi})^\beta = (w_{Z1} Z_{i1} + w_{Z2} Z_{i2}) (h_i^Z)^{-\beta}. \end{aligned} \quad (20)$$

The first order condition for an extreme point will then be

$$\frac{1}{c_i} \left\{ \alpha \gamma_i C_1 (h_i^Y)^{\alpha-1} - \beta \gamma_i C_2 (h - h_i^Y)^{\beta-1} \right\} + 2\delta_i (h_i^Y)^{-1} = 0. \quad (21)$$

We now formulate the minimum constraint using the second order derivative.

Assumption 1 (Minimum work requirement in the knowledge sector). To ensure normal behavior when h_i^Y becomes very small, assume that there is a level h_{\min} below which individuals cannot choose to work in the knowledge sector unless they choose $h_i^Y = 0$. If they want to work there, there is a minimum work requirement of h_{\min} . It is assumed that this minimum level satisfies

$$\begin{aligned} \frac{1}{c_i} \cdot (h_i^Y)^{-2} \left\{ (\alpha - 1) \alpha \cdot C_1 (h_{\min})^{\alpha-2} + (\beta - 1) \beta \cdot C_2 (h - h_{\min})^{\beta-2} (h_{\min})^2 \right\} \\ - \left(\frac{h_i^Y}{c_i} \cdot \frac{\partial c_i}{\partial h_i^Y} \right)^2 > 2 \frac{\delta_i}{\gamma_i} \quad \forall L_{Yi}, L_{Zi} \end{aligned} \quad (22)$$

and

$$U(L_{Yi}, L_{Zi}, 0) > U(L_{Yi}, L_{Zi}, h_{\min}) \quad \forall L_{Yi}, L_{Zi}. \quad (23)$$

where

$$\frac{\partial c_i}{\partial h_i^Y} = \alpha C_1 (h_i^Y)^{\alpha-1} - \beta C_2 (h - h_i^Y)^{\beta-1}. \quad (24)$$

Proposition 2 *Given assumption 1, individuals will only work in one of the sectors. We either have $h_i^Y = 0$ and $h_i^Z = h$ or $h_i^Y = h$ and $h_i^Z = 0$.*

Assumption 1 asserts that there is a minimum level of h_i^Y , denoted h_{\min} , that is such that at all higher levels individuals will choose either to only work in the knowledge sector, $h_i^Y = h$, or in the goods sector, $h_i^Y = 0$ implying $h_i^Z = h$. It follows directly from the second order derivative.

Notice that if $\delta_i = 0$, then from the first order condition we have

$$\alpha C_1 (h_i^Y)^{\alpha-1} - \beta C_2 (h_i^Z)^{\beta-1} = 0 \quad (25)$$

so that $\frac{\partial c_i}{\partial h_i^Y} = 0$, and the second order derivative is always positive. In this case, the extreme point described by the first order condition is shown to be a minimum point. There is no interior maximum solution and the maximum point will be found on the boundaries (with either $h_i^Y = h$ and $h_i^Z = 0$ or $h_i^Y = 0$ and $h_i^Z = h$).

If $\delta_i > 0$ then the first order condition implies that

$$\alpha \gamma_i C_1 (h_i^Y)^{\alpha-1} - \beta \gamma_i C_2 (h_i^Z)^{\beta-1} < 0 \quad (26)$$

and thereby $\frac{\partial c_i}{\partial h_i^Y} < 0$. The second order derivative will then be negative (implying a maximum at an interior point) if

$$\frac{1}{c_i} \cdot \left\{ (\alpha - 1) \alpha \cdot C_1 (h_i^Y)^\alpha + (\beta - 1) \beta \cdot C_2 (h_i^Z)^{\beta-2} (h_i^Y)^2 \right\} - \left(\frac{1}{c_i} \cdot \frac{\partial c_i}{\partial h_i^Y} \right)^2 (h_i^Y)^2 < 2 \frac{\delta_i}{\gamma_i} \quad (27)$$

We see that this will be the case when h_i^Y is very low (As $h_i^Y \rightarrow 0$ the left hand side of the equation goes towards zero). This is natural since in our utility function a positive δ_i implies that when $h_i^Y = 0$ the marginal utility of work in the knowledge sector is infinite.

That an individual will always want to work at least a little in the knowledge sector is an interesting aspect of the model, but in the following we wish to concentrate on the effects of specialization. We therefore make the above assumption that there is a minimum work requirement in the knowledge sector, so that it is not possible to work small amounts of hours there.

3.3 Choosing the sector in which one wishes to work

After discussing to which degree individuals specialize within a given sector and the conditions needed for them to choose to only work in one sector, I now come to the final point of describing which sector they want to work in. Individuals will choose to work in the knowledge sector if

$$U(L_{Yi}, L_{Zi}, h) > U(L_{Yi}, L_{Zi}, 0) \quad (28)$$

As before, we assume without loss of generality that $A_{Y1i} > A_{Y2i}$ and $A_{Z1i} > A_{Z2i}$. Then individuals will choose to work in the knowledge sector if their utility is higher in the knowledge sector than in the goods sector,

$$\gamma_i \ln([A_{Y1i} (hL_{Yi})^\alpha + A_{Y2i} (h - hL_{Yi})^\alpha]) + \delta \ln hL_{Yi} + \delta \ln h(1 - L_{Yi}) > \gamma_i \ln \left([A_{Z1i} (h)^\beta] \right). \quad (29)$$

Sufficient (but not necessary) conditions for choice of work are.

Proposition 3 *A sufficient condition for individuals to wish to work in the knowledge sector is that*

$$\ln \left([A_{Z1i} (h)^\beta] \right) < \ln \left([A_{Y1i} + A_{Y2i}] \left(\frac{h}{2} \right)^\alpha \right) + 2 \frac{\delta}{\gamma_i} \ln \frac{h}{2} \quad (30)$$

while a sufficient condition for them to wish to work in the goods sector is that

$$\ln ([A_{Y1i} (h)^\alpha]) < \ln [A_{Z1i} (h)^\beta] - 2 \frac{\delta}{\gamma_i} \ln \frac{h}{2}. \quad (31)$$

If neither of these conditions are met, the matter must be examined further.

Proof. Easy to see from the condition above, noting that in the interval $L_{Yi} \in \langle \frac{1}{2}, 1 \rangle$ the utility of working in the knowledge sector, $\gamma_i \ln([A_{Y1i} (hL_{Yi})^\alpha + A_{Y2i} (h - hL_{Yi})^\alpha]) + \delta \ln hL_{Yi} + \delta \ln h(1 - L_{Yi})$, has a minimum at $L_{Yi} = \frac{1}{2}$, while the highest value of $(\ln L_{Yi} + \ln(1 - L_{Yi}))$ occurs for $L_{Yi} = \frac{1}{2}$. ■

Proposition 3 is mainly illustrative, giving two cases where the choice of sector is easily determined. We see that an individual will choose to work in the goods sector if the pecuniary reward for choosing to work there over working in the knowledge sector is larger than the joy derived directly from working in that sector. If an individual derives no direct utility from working in the knowledge sector $\delta = 0$, then she or he earns more in the goods sector than in the knowledge sector and will choose that sector.

4 A simple example

The model I have presented is very simple, but with a need to handle the effects of increasing returns with care. From a few plausible assumptions about some types of work giving direct utility and there being increasing returns to most work, we have established results enabling us to replicate the table presented in the introduction.

Assume four types of individuals endowed with either the low talent vector, $t_l = (0.8, 0.8, 0.8, 1)$, or the high talent vector $t_h = (1.2, 1.2, 1.2, 1)$ and either a positive utility $\left(\frac{2\delta}{\gamma} = \frac{1}{4}\right)$ or zero utility ($\delta = 0$) from working in the knowledge sector. This gives us four combinations

Table 2. Combinations of talent and interest

	interest in knowledge	
	low, $\delta = 0$	high, $\delta > 0$
low talent, t_l	ll	lh
high talent, t_h	hl	hh

Notice that all individuals have equal talents in the production of Z_{i2} . This can be thought of as the tasks an academic undertakes in industry (see table 1), while production of Z_{i2} can be viewed as management in the industrial sector. We can also think of Y_{i1} and Y_{i2} as respectively teaching and doing research in a government institution. We arbitrarily assume the following variables

Table 3. Parameter values

h	h_{\min}	α	β	w_{Y1}	w_{Y2}	w_{Z1}	w_{Z2}
2000 hrs	600 hrs	1,35	1,65	9	9	2	2

The minimum number of hours one must work in the government sector (if one wishes to work there) is 30% of available time. There are differing increasing returns to scale in industry and government, where the piece rate wages are used to calibrate the different functions to give comparable incomes. These wage rates are not equilibrium rates in the sense that they equilibrate supply and demand. They are examples of wages that will lead the four types of individuals in to three different job situations with four different incomes.

The results from using the above parameters and wages is that the four types of individuals divide into four work group, as shown in the table below. The individuals who have no direct utility of work, have income maximizing jobs in the private sector. Those with such a direct utility choose to earn less, due to the extra job satisfaction they then get. High talent individuals earn more than those with less talent. In industry the high talented become managers, while the low talented take a lower paid academic position. In government, the high talent and low talent individuals specialize to the same degree. In our example the parameters are symmetrical between the two tasks, so individuals are indifferent to which task they specialize in (workers work 84% of the time on one task, for example teaching, and 16% of the time on the other).

Table 4. Simulated monthly compensation in Norwegian kroner

	Labor Income
Industry	
- <i>management</i> , high talent, low interest	60 600
- <i>academic position</i> , low talent, low interest	37 300
Central Government	
- <i>academic position</i> , high talent, high interest	27 097
- <i>academic position</i> , low talent, high interest	16 700

The part of the pay difference due to the individuals in government not fully specializing is 3900 NOK in the case of the high talent individuals and NOK 2400 in the case of the low talent individuals. From Table 4 we see that income does not necessarily correlate with opportunities. High talent individuals have by definition more opportunities than low talent individuals. Even so, the low talent individual working in industry earns more than the high talent worker in government. This can be seen as example of how evaluating individual welfare by outcomes is not the same as evaluating them by their opportunities, as argued by Sen (1985) and Nussbaum (2000).

5 Consequences for the firm

In the above formulation, the workers supply efficiency units of labor and receive the marginal product of these efficiency units as their wage, leading to the workers capturing the efficiency gains from specialization. This formulation requires that the efficiency of the worker is measurable and that the firms allow the workers themselves to choose the degree of specialization. There are obvious agency problems with this formulation and letting the workers choose their degree of specialization can be costly for the firm. If the workers choose to specialize the firm needs to hire fewer workers (since they are more efficient than generalist) and the matching of workers with tasks is simple.

The profit of the firm is given by

$$\Pi = \left(pX - \sum_i w_{Y1}Y_{i1} - \sum_i w_{Y2}Y_{i2} - \sum_i w_{Z1}Z_{i1} - \sum_i w_{Z2}Z_{i2} - K \right), \quad (32)$$

where K denotes all costs other than labor costs. To simplify the discussion we assume that there four types of individuals, as in the example above. Denote individuals with high talent, high interest as type 1, with low talent, high interest as type 2, high talent, low interest as type 3 and finally low talent, low interest as type 4. Let n_i denote the number of workers of type i and assume that, as in the example above, the high interest individuals choose to be generalists, while the low

interest individuals choose to be specialists. Then the profit equation can be written

$$\Pi = pX - w_{Y1}(n_1Y_{11} + n_2Y_{21}) - w_{Y2}(n_1Y_{12} + n_2Y_{22}) - w_{Z1} \cdot n_3Z_{11} - w_{Z2} \cdot n_4Z_{22} - K. \quad (33)$$

There are n_1 type 1 workers delivering Y_{11} units of efficiency labor units in task 1 and Y_{12} units of efficiency labor units in professional task 1. Workers of type 2 distribute their labor similarly, while workers of type 3 specialize at non-professional task 1 and type 4 workers specialize at non-professional task 2. Given wages set in a competitive market, maximizing profits leads to familiar first order conditions (we get four first order equations in four variables n_1, n_2, n_3 and n_4).

The situation becomes more complicated if there are many more tasks and if the talents of workers vary more (for example with a continuum of different talent parameters). The optimization problem for the workers who specialize remains simple, but employing the right amount of generalists of different abilities and interests so that tasks get done in an optimal manner becomes much more complex. Every time relative wages change, generalists will change how much they work on the different tasks and the firm must recalibrate it's match of workers and tasks. When there are many tasks, it also becomes doubtful whether there will be observable wages for each separate task.

To deal with the complexities of many tasks and differing abilities among workers, the firm may instead offer a finite set of jobs, where each job implies a combination of tasks to be done and a wage for doing these tasks. In this case the workers cannot freely choose how much to work on each task. They must choose the job which optimizes their combined desire for income and for generalization. Competition in the labor market for generalists will then be a market where there is a supply and demand of jobs, each job being a set of tasks and a wage. The model above can still be seen as an approximation of the choice faced by the workers, only, instead of choosing directly how much to work on different tasks, the workers must choose between sets of jobs. If the jobs available are many and diverse it should be possible for the worker to find a combination of tasks and wage which is close to what the worker would wish for if she could choose tasks freely.

If the firm does not face extra costs from letting the workers choose their degree of specialization, it is difficult to constrain the workers ability to do so, since it is the worker herself who "pays" for possibility of being a generalist (by accepting a lower income). On the other hand if there are such costs, it becomes possible for the firms to constrain the workers. Let us for example introduce a constant administrative cost, τ , for each individual working on each task, so the profit equation (33) becomes:

$$\begin{aligned} \Pi = pX - w(n_1Y_{11} + n_2Y_{21}) - w(n_1Y_{12} + n_2Y_{22}) - w_{Z1} \cdot n_3Z_{11} - w_{Z2} \cdot n_4Z_{22} - K \\ - 2(n_1 + n_2)\tau - n_3\tau - n_4\tau, \quad (34) \end{aligned}$$

when all the workers can choose to be generalists and wages are assumed to be equal in the two professional tasks, $w = w_{Y1} = w_{Y2}$ (as in the example above). The marginal productivity of adding an extra worker of type 1 will in this case be

$$p \frac{\partial X}{\partial n_1} = w \cdot (Y_{11} + Y_{12}) - 2\tau. \quad (35)$$

If workers of type 1 and type 2 are forced to specialize with type 1 workers specializing in task 1 and the type 2 workers in task 2, the profit equation becomes:

$$\Pi = pX - w \cdot (n_1 Y_{11}^* - n_2 Y_{22}^*) - w_{Z1} \cdot n_3 Z_{11} - w_{Z2} \cdot n_4 Z_{22} - K - (n_1 + n_2 + n_3 + n_4) \tau, \quad (36)$$

where the variable Y_{ji}^* denotes the units of efficiency labor delivered by type i workers when specializing in task j . In this case the marginal productivity becomes

$$p \frac{\partial X}{\partial n_1} = w \cdot Y_{11}^* - \tau. \quad (37)$$

The difference in marginal productivity is $w \cdot (Y_{11}^* - (Y_{11} + Y_{12})) + \tau$. If the costs, τ , are high enough, the firm can require all workers to specialize, knowing that the costs of employing generalists will keep other firms from offering competitive generalist positions to their workers.

At the beginning of the paper it was hypothesized that some of the observed differences in incomes can be due to some workers sacrificing high wages to be able to do work they find enjoyable. Having described how the workers might choose jobs if work brings positive utility, the paper now ends with the observation that there might not be many firms that will give workers this possibility. In a more dynamic framework, one might speculate that firms might offer low salaried, entry-level employees the opportunity to be generalists (revealing their talent and interest vectors), but require specialization when they advance into higher wage positions. In the above example, management is associated with specialization, but it could be argued that management is the ultimate type of generalist work. Advancing into management from a career of increasing specialization can then be seen as escaping from the frustration of a one-dimensional work burden.

6 Conclusions

The simple model presented in the paper describes the trade-offs workers make when they receive utility from working different tasks but with increasing returns to concentrating on one. While the model has very simple functional forms, the results mainly depend on the combination of a concave utility function (so worker wish to do many things) and the increasing returns attained by concentrating on only one task.

In the model individuals with small opportunity sets (in our terminology low talent individuals) may earn more than those with large opportunity sets. This has consequences for taxation policy

and welfare analysis. For the firms it can be costly to let workers choose their combination of tasks freely. If so they might choose to only offer certain combinations of wages and tasks. If the costs of letting workers be generalists is large, there might not be jobs on offer which allow a worker to be a generalist.

References

- [1] Bhaskar, V., To, T., 2003. Oligopsony and the distribution of wages. *European Economic Review* 47, 369-394.
- [2] Blackburn, M., Neumark, D., 1992. Unobserved ability, efficiency wages, and interindustry wage differentials. *Quarterly Journal of Economics* 107, 1421-1436.
- [3] Cheng, W., Yang, X., 2004. Inframarginal analysis of division of labor. A survey. *Journal of Economic Behavior & Organization* 55, 137-174.
- [4] Diamantaras, D., Gilles, R.P., 2004. On the microeconomics of specialization. *Journal of Economic Behavior & Organization* 55, 223-236.
- [5] Gibbons, R., Katz, L., 1992. Does unmeasured ability explain inter-industry wage differentials? *Review of Economic Studies* 59, 515-535.
- [6] Mani, A., Mullin, C.H., 2004. Choosing the right pond: social approval and occupational choice. *Journal of Labor Economics* 22, 835-861.
- [7] Nussbaum, M.C., 2000. *Women and Human Development: The Capabilities Approach*. Cambridge University Press, Cambridge.
- [8] Sen, A.K., 1985. *Commodities and Capabilities*. Oxford University Press, Oxford.
- [9] Smith, A. 1776. *An inquiry into the Nature and Causes of the Wealth of Nations*. University of Chicago Press, Chicago (reprint).
- [10] Young, A.A., 1928. Increasing returns and economic progress. *The Economic Journal* 38,527-542.

Appendix A.

Proposition 1. *Workers wishing to work in the technology sector ($h_i^Y = h$ and $h_i^Z = 0$) will choose to work at both tasks, but working longer hours at the task with the highest talent adjusted wage A_{Yki} . The the first order condition*

$$\frac{\delta_i}{\gamma_i \alpha} - \left(\frac{2\delta_i}{\gamma_i \alpha} + 1 \right) L_{Yi} + \frac{w_{Y1} Y_{i1}}{w_{Y1} Y_{i1} + w_{Y2} Y_{i2}} = 0$$

applies. Workers wishing to work in the goods sector ($h_i^Y = 0$ and $h_i^Z = h$) will choose a specialization strategy, only working at the task with the highest talent adjusted wage A_{Zki} .

Proof. The first order condition is an implicit equation in L_{Yi} which we cannot explicitly solve, except in special circumstances. Even so, it is fairly simple to analyze by decomposing it into two parts

$$-V_1(L_{Yi}) + V_2(L_{Yi}) = 0$$

where

$$\begin{aligned} V_1(L_{Yi}) &= \left(\frac{2\delta_i}{\gamma_i \alpha} + 1 \right) L_{Yi} - \frac{\delta_i}{\gamma_i \alpha} \\ V_2(L_{Yi}) &= \frac{w_{Y1} Y_{i1}}{w_{Y1} Y_{i1} + w_{Y2} Y_{i2}}. \end{aligned}$$

Without loss of generality we assume that $A_{Y1i} \geq A_{Y2i}$, so that $L_{Yi} \in [\frac{1}{2}, 1]$ and $V_2(L_{Yi}) \geq \frac{1}{2}$.

The decomposition of the first order condition into two parts is done to help in determining that there is an interior solution $\langle \frac{1}{2}, 1 \rangle$ to the maximization problem (if $A_{Y1i} \geq A_{Y2i}$ then $L_{Yi} \in [\frac{1}{2}, 1]$). The $V_1(L_{Yi})$ function is linear in L_{Yi} . There will be a unique solution to the first order condition if the $V_2(L_{Yi})$ function is concave and starts below the $V_1(L_{Yi})$ function (at $L_{Yi} = \frac{1}{2}$), then increases monotonously, ending at a higher end point than $V_1(L_{Yi})$ (at $L_{Yi} = 1$). The concavity of $V_2(L_{Yi})$ assures us that the two functions cross only once.

The second order derivative is found to be

$$\begin{aligned} \frac{\partial U^2 U_Y(L_{Yi}, h)}{(\partial L_{Yi})^2} &= \left\{ -\frac{\partial V_1(L_{Yi})}{\partial L_{Yi}} + \frac{V_2(L_{Yi})}{\partial L_{Yi}} \right\} \frac{\alpha \gamma_i}{L_{Yi} - (L_{Yi})^2} \\ &\quad + \{-V_1(L_{Yi}) + V_2(L_{Yi})\} \frac{\alpha \gamma_i}{(L_{Yi} - (L_{Yi})^2)^2} (1 - 2L_{Yi}) \end{aligned}$$

which at the point given by the first order condition becomes

$$\frac{\partial U^2(L_{Yi}, \cdot, h)}{(\partial L_{Yi})^2} = \left\{ -\frac{\partial V_1(L_{Yi})}{\partial L_{Yi}} + \frac{V_2(L_{Yi})}{\partial L_{Yi}} \right\} \frac{\alpha \gamma_i}{L_{Yi} - (L_{Yi})^2},$$

with

$$\begin{aligned}\frac{\partial V_1}{\partial L_{Y_i}} &= \frac{2\delta_i}{\gamma_i\alpha} + 1 \\ \frac{\partial^2 V_1}{(\partial L_{Y_i})^2} &= 0\end{aligned}$$

$$\begin{aligned}\frac{\partial V_2}{\partial L_{Y_i}} &= \frac{\alpha}{L_{Y_i}(1-L_{Y_i})} \cdot \frac{w_{Y_2}Y_{i2}}{w_{Y_1}Y_{i1}} \cdot \left(\frac{w_{Y_1}Y_{i1}}{w_{Y_1}Y_{i1} + w_{Y_2}Y_{i2}}\right)^2 > 0 \\ \frac{\partial^2 V_2}{(\partial L_{Y_i})^2} &= 2\alpha^2 \cdot A_{Y_{1i}}A_{Y_{2i}} \cdot \frac{(L_{Y_i}(1-L_{Y_i}))^{\alpha-2}}{(w_{Y_1}Y_{i1} + w_{Y_2}Y_{i2})^2} \left[\left(L_{Y_i} - \frac{1}{2}\right) - \alpha \cdot \left(\frac{w_{Y_1}Y_{i1}}{w_{Y_1}Y_{i1} + w_{Y_2}Y_{i2}} - \frac{1}{2}\right) \right]\end{aligned}$$

Furthermore we then have that

$$\begin{aligned}V_1\left(\frac{1}{2}\right) &= \left(\frac{2\delta}{\gamma_i\alpha} + 1\right) \frac{1}{2} - \frac{\delta_i}{\gamma_i\alpha} = \frac{1}{2} \\ V_2\left(\frac{1}{2}\right) &= \frac{1}{1 + \frac{A_{Y_{2i}}}{A_{Y_{1i}}}\left(\frac{1-\frac{1}{2}}{\frac{1}{2}}\right)^\alpha} = \frac{1}{1 + \frac{A_{Y_{2i}}}{A_{Y_{1i}}}} \geq \frac{1}{2}\end{aligned}$$

and

$$\begin{aligned}V_1(1) &= \left(\frac{2\delta}{\gamma_i\alpha} + 1\right) 1 - \frac{\delta_i}{\gamma_i\alpha} = 1 + \frac{\delta_i}{\gamma_i\alpha} \\ V_2(1) &= \frac{1}{1 + \frac{A_{Y_{2i}}}{A_{Y_{1i}}}\left(\frac{1-1}{1}\right)^\alpha} = 1\end{aligned}$$

In other words, for all possible values of L_{Y_i} , given that $A_{Y_{1i}} \geq A_{Y_{2i}}$, we have that $V_1\left(\frac{1}{2}\right) < V_2\left(\frac{1}{2}\right)$ and $V_1(1) > V_2(1)$. Since they are two continuous increasing functions in L_{Y_i} , they must be equal at least at one point on $L_{Y_i} \in \left[\frac{1}{2}, 1\right]$. Everything is symmetrical between task 1 and 2 except that the talent adjusted wage $A_{Y_{ki}}$ is higher in 1 than in 2. There will then be no interest in specializing at task 2, since a higher income will be achieved by specializing at task 1 (the utility of hours spent at the two tasks is equal).

If V_2 is concave there can only be one solution where $V_1(L_{Y_i}) = V_2(L_{Y_i})$ and thereby that $\frac{\partial U}{\partial L_{Y_i}} = 0$. The function V_2 will be concave $\left(\frac{\partial^2 V_2}{(\partial L_{Y_i})^2} < 0\right)$ if

$$L_{Y_i} - \frac{w_{Y_1}Y_{i1}}{w_{Y_1}Y_{i1} + w_{Y_2}Y_{i2}} < 0,$$

which must be the case because rearranging this gives us

$$1 < \frac{A_{Y_{1i}}}{A_{Y_{2i}}} \left(\frac{L_{Y_i}}{1-L_{Y_i}}\right)^{\alpha-1},$$

since where $A_{Y_{1i}} > A_{Y_{2i}}$ and $L_{Y_i} > 1 - L_{Y_i}$ by assumption. For $A_{Y_{1i}} = A_{Y_{2i}}$, there are two solutions, one as given above and the other for $L_{Y_i} = \frac{1}{2}$. In such a case one must check to see which gives the maximum utility.

In the case where individuals wish to work in the goods sector the reasoning is similar to the above giving us the first and second order conditions

$$\frac{\partial U(\cdot, L_{Zi}, 0)}{\partial L_{Zi}} = \beta \gamma_i \frac{1}{c_i} \left(A_{Z1i}(h)^\beta (L_{Zi})^{\beta-1} - A_{Z2i}(h)^\beta (1 - L_{Zi})^{\beta-1} \right) = 0$$

and

$$\frac{\partial U^2(\cdot, L_{Zi}, 0)}{(\partial L_{Zi})^2} = (\beta - 1) \beta \gamma_i \frac{1}{c_i} \left(A_{Z1i}(h)^\beta (L_{Zi})^{\beta-2} + (\beta - 1) A_{Z2i}(h)^\beta (1 - L_{Zi})^{\beta-2} \right) > 0$$

implying only a minimum point. An individual wishing to work in the goods sector therefore works at only one task. ■