ISSN: 2039-4004

department of economics
"S. Cognetti de Martiis"

Via Po, 53 – 10124 Torino (Italy)
Tel. (+39) 011 6704043 - Fax (+39) 011 6703895
URL: http//www.de.unito.it

WORKING PAPER SERIES

A microeconometric-computational approach to empirical optimal taxation: outline of a project

Ugo Colombino

Dipartimento di Economia "S. Cognetti de Martiis"

Working paper No. 18/2012

Università di Torino

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Ugo Colombino
Department of Economics Cognetti De Martiis, University of Turin, Italy

Summary

Optimal taxation (OT) is a highly sophisticated theoretical construct with potential implications on many crucial policy issues such as the progressivity of the marginal tax rates on personal incomes, the definition of the tax base, the design of income support mechanisms and family-related transfers etc. Many authors have explored the empirical and policy implications of OT theory starting from the seminal contribution by Mirrlees (1971). A first group of contributions in the period 1971 – 1989 mainly consist of illustrative numerical exercises rather than empirical applications. The optimal tax-transfer regime that most commonly emerges from these efforts is a negative income tax + a (almost) flat tax. More recently, a second group of studies are more focussed on policy implementations and, relying on extensions or reformulations of Mirrlees's model, attempt to establish a closer link between theory and data or econometric estimates. We argue that both generations of studies suffer from taking for granted that the solution to the optimal taxation problem must be an analytical one, i.e. a "formula" to be "calibrated" with numerical guesses or estimates. As a first consequence, the theoretical models must adopt very restrictive assumptions in order to generate analytical solutions: recent studies suggest that the policy prescriptions – the optimal tax-transfer rules – are not robust with respect to alternative assumptions. As a second consequence, the theoretical results are potentially inconsistent with the empirical estimates that are typically generated under different (much less restrictive) assumptions. This paper illustrates a project where we pursue a different approach. First, the behaviour of economic agents is represented by flexible microeconometric models that can account for complex details in the preferences and in the constraints, thus avoiding most of the restrictive assumptions adopted in the theoretical literature: thus we can expect the policy implications to be more robust and realistic. Second, optimal tax-transfer rules are obtained computationally, by iteratively simulating a microeconometric model until the social welfare criterion is maximized. Third, by replicating the microeconometric–computational exercise on different country-specific datasets, we can identify the mapping from the primitives of the economy and the characteristics of the optimal tax-transfer rule, thus attaining a level of generality close to the one attained by OT theory.

1. Background

Many authors (Atkinson (1972), Tuomala (1984, 1990), Diamond (1998), Saez (2001) etc.) have explored the empirical and policy implications of the conditions established by Mirrlees (1971) for the optimality of the personal income tax-transfer rule.

We can distinguish two "generations" of studies. The first (Appendix 1), from 1971 up to the late '80s, mainly consists of illustrative numerical exercises rather than of empirical applications. These exercises have been mostly disappointing for those – probably including Mirrlees himself – who expected an optimal profile of monotonically increasing marginal tax rates. In fact, the results tend to converge to the following optimal tax-transfer rule:

- a lump-sum (positive) transfer;
- very high (and declining) marginal tax rates on low incomes up to a break-even point (where gross income is equal to the transfer);
- constant or slightly increasing marginal tax rates beyond the break-even point.

If we translate the above scenario into a policy reform, by and large it corresponds to a Negative Income Tax + a (almost) Flat Tax (NIT + FT).

More or less egalitarian Social Welfare functions imply different level of the transfer and of the average tax but do not significantly change the marginal tax rates profile.

Up to recent years, most researchers working in the optimal taxation area tended to conclude that the above scenario was the definitive lesson to be drawn from Mirrlees's model. If one was looking for a model envisaging a progressive profile of increasing marginal tax rates, it had to be searched for outside that model. Recent contributions by Tuomala (2006, 2010) however, convincingly argue that the results recalled above are very likely to be forced by too restrictive or too unrealistic specifications chosen for the numerical simulation. In particular, it appears that typical assumptions such as: a constant elasticity of labour supply, a constant (or missing) income effect, common utility specification such as the Cobb-Douglas, the CES, or the quasi-linear, and productivity distributions such as the log-normal, all tend to favour something close to the NIT + FT scenario. Tuomala (2010) presents a numerical illustration that adopts a much more flexible specification for the utility function, namely a quadratic form. As a first consequence, he gets a pattern of heterogeneous (with respect to income levels) elasticities and income effects which closely matches the empirical evidence. Moreover he obtains an optimal solution that envisages a lump-sum transfer and a profile of monotonically increasing marginal tax rates, i.e. something that resembles a Universal Basic Income + a Progressive Tax (UBI + PT).

The second generation (Appendix 2) is characterized by a more definite focus on the policy implementation of the optimal taxation theory. A series of contributions have reformulated or generalized Mirrlees's results in order to express them in terms of empirically measurable concepts, e.g. labour supply elasticities (Revesz, 1989; Saez, 2001) and establish a more direct link between OT theory and the design of policy reforms. As a corollary of the more policy-oriented position, these contributions use real datasets and econometric estimates instead of simple "calibration" procedures (as was the case with the First Generation studies). Saez (2002) develops a discrete model that assigns a crucial role to the relative magnitude of the labour supply elasticities at the extensive and at the intensive margin. This framework turned out to be very influential in the last few years. So far we know four applications of this model: Blundell et al., 2009 (optimal taxation of single mothers in Germany and UK); Haan et al., 2007 (optimal design of *children benefits* in Germany); Immervoll et al., 2007 (evaluation of income maintenance policies in European countries); Brewer et al., 2008 (report prepared for the Mirrlees Commission for tax reform in the UK). In all four applications the model tends to attest the superiority of mechanisms such as in-work benefits (i.e. rules involving negative marginal taxes, or subsidies on the wage rate) over alternatives like NIT or UBI. It might be the case that the simplifications adopted by the model somehow force the result. Since the role of the two types of elasticities is so predominant and since the empirical participation elasticity tends to be much larger than the intensive-margin elasticity, it is not so surprising that the optimal rule turns out to be one envisaging subsidies on low wages rather than transfers to low income people. Nonetheless it is important that the model adds a new scenario (in-work benefits) as an alternative to what appeared to be the inescapable consensus – i.e. the NIT + FT – in the first generation of studies.

2. A critique

The studies belonging to the First and Second generations have much in common as to the type of solution they aim at and as to the relationship between the theoretical solution and the empirical evidence (Colombino, 2009). More precisely:

a) The researcher looks for an analytical solution to the optimal taxation problem, i.e. a "formula" that allows to compute the optimal taxes or marginal tax rates as function of exogenous variables and parameters.
b) The numerical simulations consist in calculating the analytical solution with exogenous variables and parameters assigned numerical values produced by "educated guesses" (first generation) or econometric estimates (more often in the second generation).

Analytical solutions are very useful to understand (and teach) the "grammar" of the problem. But when it comes to evaluating or designing actual tax-transfer rules, they might have undesirable implications. First, in order to get an analytical solution we are forced to adopt many simplifying and restrictive assumptions. Second, when we "feed" the formulas with empirical measures, we are very likely to face an inconsistency between the theoretical results and the empirical evidence, since the latter was typically generated under assumptions that are very different for those that made it possible obtaining the former. The paper by Blundell et al. (2009) represents an example of this potential inconsistency. That study adopts the model of Saez (2002) and in particular it uses expression (2.4) of Appendix 2 by assigning numerical values to the elasticities derived from estimates obtained with a detailed microeconometric model. However, Saez (2002) assumes there are no income effects and specifies a very special and limited representation of choices at the intensive margin. None of these assumptions are shared by the microeconometric model used to measure the elasticities. The microeconometric model is much more flexible, it accounts for income effects and implies very different responses at the intensive margin. Therefore we might expect that the elasticities produced by the microeconometric model do not fit well into the framework of Saez (2002).

3. A new (complementary) approach

Modern microeconometric models of labour supply are based on very general and flexible assumptions. They can accommodate many realistic features such as general structures of heterogeneous preferences, simultaneous decisions of household members, non-unitary mechanisms of household decisions, complicated (non-convex, non continuous, non-differentiable etc.) constraints and opportunity sets, multidimensional heterogeneity of both households and jobs, quantitative constraints etc. It is simply not feasible (at least so far) to obtain analytical solutions for the optimal taxation problem in such environments. Yet those features are very relevant and important especially in view of evaluating or designing reforms (Colombino, 2009). We propose an alternative (or maybe complementary) procedure, which consists of using a microeconometric model to obtain a computational solution of the optimal taxation problem (Appendix 3). Analytical solutions still remain crucially important in suggesting promising classes of tax-transfer systems that can then be more deeply investigated with the microeconometric model. The latter, which primarily simulates the agents' choices by utility maximization, is embedded into a global maximization algorithm that solves the social planner's problem, i.e. the maximization of a social welfare function subject to the public budget constraint. Besides the flexibility in representing agents' preferences and complex opportunity sets, the microeconometric-computational also allow us to account for three issues that in OT theory are mostly ignored or treated under very special assumptions: (a) the interpersonal comparability of the preferences to be aggregated into a social welfare criterion; (b) the implications of different definitions of the social welfare criterion; (c) accounting for market equilibrium (Colombino 2012).

The examples so-far available of the computational approach to empirical OT, however, are indeed example-dependent. A different dataset (a different country, the same country in a different year, a different demographic composition etc.) are likely to produce different results. It seems that by adopting the computational approach, we have been able to avoid too restrictive assumptions at the price of loosing generality. On the contrary, the analytical results of OT theory establish a precise and general relationship between optimal tax rates and characteristics of the economy such as the distribution of productivity, labour supply elasticity etc. Can we attain something similar within the computational approach? In principle the answer is affirmative. This line of research is the main focus of the present project. We can compute optimal taxes on different economies, and then investigate the relationship between the characteristics (the primitives) of those economies and the corresponding optimal taxes. The objects to be put in relation are complex objects (structured clusters of variables and parameters), so the analysis requires a version of generalized comparative statics. The appropriate techniques are becoming available for this purpose.[1]

4. Relationship with other literatures

The philosophy inspiring the approach described in Section 2.3 is similar to the one adopted since long ago in engineering and recently and successfully also in other applications of mechanism design (auctions, negotiation procedures, matching markets etc.: Roth (2002) provides a very inspired survey. The analytical solution is complemented by computational experiments that account for a host of realistic features that cannot be included in the theoretical model: "*Consider the design of suspension bridges. The simple theoretical model in which the only force is gravity, and beams are perfectly rigid, is elegant and general. But bridge design also concerns metallurgy and soil mechanics, and the sideways forces of water and wind. Many questions concerning these complications can't be answered analytically, but must be explored using physical or computational models. These complications, and how they interact with the parts of the physics captured by the simple model, are the domain of the engineering literature.*" (Roth, 2002).

In a similar vein, P. Milgrom – a leader both in optimal auction theory and in practical design and implementation of auctions – argues in favour of an approach that complements game-theoretic principles with empirical evidence and computational tools in designing effective auction protocols: "*Besides the very demanding behavioral assumptions that characterize the theoretical mechanism design approach, the formal models of the theory typically capture only a small subset of the issues that a real auctioneer faces. [Many] important issues…are usually omitted from mechanism design models ... While none of these is incompatible with mechanism design theory in principle, accounting for all in a single optimization model is beyond the reach of present practice.*" (Milgrom 2002).

[1] We refer for example to the techniques pioneered by P. Milgrom and J. Roberts for the analysis of organizational structures, e.g. Milgrom and Roberts (1994).

OT is an application of mechanism design theory. Other applications such as auctions, negotiations and matching mechanisms in the last decades have been an area of successful implementations due to a creative mix of theory and computational tools, actually opening a new domain of applied research referred to as Computational Mechanism Design.[2] This line of research may provide a blueprint for a similar approach in OT.

5. Organization of the project

5.1. Estimation of microeconometric models of household labour supply and identification of optimal tax-transfer rules.

The background is represented by a series of papers where the microeconometric-computational approach to optimal taxation is adopted (Appendix 3). Aaberge and Colombino (2013, 2006) identify optimal taxes for Norway within the class of 9-parameter piece-wise linear tax-transfer rules. Aaberge and Colombino (2012) perform a similar exercise for Italy. These papers also address the issue of interpersonal comparability and explore the implications of alternative social welfare criteria. Aaberge and Flood (2009) study the design of tax-credit policies in Sweden. Blundell and Shepard (2010) focus on the optimal tax-transfer systems for lone mother in the UK. Colombino et al. (2010) study the design of income support mechanisms in various European countries. In our project we will pursue the development of models appropriate for a sufficiently large set of countries and the identification of more general tax-transfer rules. This step requires very powerful computing resources, advanced methods for non-standard optimization problems, a large and comparable database and flexible and reliable tax microsimulation algorithms. As far as data and microsimulation are concerned, the resources available through EUROMOD are ideal candidates. In principle we should be able to use all the EUROMOD country-specific datasets plus Norway and possibly some Eastern European countries.

5.2. Identification of the mapping [primitives of the economy] → [optimal tax-transfer rule] through statistical methods and robust comparative statics.

In general one might want to know how a certain 'primitive' of the economy (e.g. the inequality in the distribution of the skills) affects a certain characteristic of the optimal tax-transfer rule (e.g. the degree of progressivity or the type of income support). In classical optimization theory the answer would be given by comparative statics tools. However in our case we are not in a classical environment. As explained in Appendix 3, what we are dealing with is the maximization of a social welfare function whose arguments are the optimized utility levels of the households; the social choice variables are the characteristics of the tax-transfer rule, which might also be discrete variables (such as the choice among individual and joint taxation); the households face opportunity sets that might be non-convex, discrete etc.; summing up, the problem lacks all the nice properties of convexity, differentiability, divisibility etc. that are required by the standard methods of comparative statics. Recently, new

[2] Surveys of Computational Mechanism Design are provided by Hyafil (2005), Parkes (2002).

methods have been made available that do not require those special properties and state robust conditions under which certain answers to the above questions can be given (Robust Comparative Statics).[3] Whether these conditions hold or not can be tested empirically once we have obtained optimal rules for a sufficiently large set of economies.

[3] An introduction to Robust Comparative Statics is provided by Gans (1996). See also Athey et al. (1998, 2002).

Appendix 1

We start by recalling the basic model introduced by Mirrlees (1971) – in a version with preferences separable in income and effort.

Agents – households – differ only in their market productivity *w*.

An agent with productivity *w* solves

$$
(1.1) \qquad \begin{array}{l} \max_{c,h} u(c,h) = U(c) - V(h) \\ \text{c.v.} \\ c = wh - T(wh) \end{array}
$$

where

w = market productivity (e.g. wage rate)

c = net available income

h = hours of work (or, more generally, effort)

$T(z)$ = tax to be paid by an agent with income $z = wh$

The Social Planner solves

$$
(1.2) \qquad \begin{array}{l} \max_{c,h} \int_0^\infty W\left(u^w\right) f(w) dw \\ \text{s.t.} \\ \int_0^\infty T\left(z^w\right) f(w) dw = R \\ u^w = \max_{c,h} u(c,h) \text{ c.v. } c = wh - T(wh) \end{array}
$$

where

$z^w \equiv wh^w$

$h^w = \arg\max_h u(c,h)$ c.v. $\text{c} = wh - T(wh)$

$f(w) = F_w(w)$ = probability density function of w

The social planner knows the distribution *F(n)* but not the individual values of *n* (or at least she is not willing – or allowed – to use them). The tax *T* only depends on *wh* (second-best solution).

By solving (1.2) we get:

$$(1.3) \qquad \frac{T_z(z^w)}{1-T_z(z^w)} = \left[\frac{1+\varepsilon(w)}{\tilde{\varepsilon}(w)}\right] \times \left[\frac{(1-F(w))}{wf(w)}\right] \times \left[\frac{U_c^w \int_n^\infty (1-g^m)(1/U_c^m) f(m)dm}{1-F(w)}\right]$$

where $g^m \equiv \dfrac{W_{u^m} U_c^m}{\lambda}$

$\varepsilon(w) =$ elasticity of the supply of h with respect to the net wage,

$\tilde{\varepsilon}(w) =$ compensated elasticity of the supply of h with respect to the net wage

and λ is the multiplier associated to the public budget constraint (= social value of public funds), so that g^w is the social marginal value (relative to public funds) of a *w*-agent's income.

In expression (1.3), in general Y^w denotes a function *Y* evaluated at the optimal choice made by an agent with productivity *w*, Y_s^w denotes the first derivative of Y^w with respect to some variable *s*.

Expression (1.3) together with the public budget constraint also determines the amount of a lump-sum transfer (positive or negative).

Numerical examples often assume there are no income effects. In this case (using the normalization $U_c^m = 1$) we get the much simpler result:

$$(1.4) \qquad \frac{T_z(z^w)}{1-T_z(z^w)} = \left[1+\frac{1}{\varepsilon(w)}\right] \times \left[\frac{(1-F(w))}{wf(w)}\right] \times \left[\frac{\int_n^\infty \left(1-\frac{W_{u^m}}{\lambda}\right) f(m)dm}{1-F(w)}\right]$$

Appendix 2

Revesz (1989), Diamond (1998) and Saez (2001), among others, present reformulations of Mirrlees's model more directly interpretable in terms of empirically observable variables. Saez (2002) develops a discrete model that assigns a crucial role to the relative magnitude of the labour supply elasticities at the extensive and at the intensive margin. The framework proposed by Saez (2002) turned out to be very influential in the last few years and we illustrate it with more details.

There are J + 1 types of job, each paying (in increasing order) $z_0, z_1, ...z_J$. Job "0" denotes a non-working conditions (non-participation or unemployment).

Net available income on job j is

(2.1) $$c_j = z_j - T_j$$

where T_j is the tax paid at income level z_j .

Each agent is characterized by one of the potential incomes $z_0, z_1, ...z_J$ and if she decides to work she is allocated to the corresponding job.

The agent of type j decides to work if $c_j \geq c_0$.

The extensive margin (or participation) elasticity is defined as:

(2.2) $$\eta_j = \frac{c_j - c_0}{\pi_j} \frac{\partial \pi_j}{\partial \left(c_j - c_0\right)}$$

where π_j is the proportion of agents on job of type j.

Working agents can also move to a different job if income opportunities change, but the movements (for reasons implicit in the assumptions of the model) are limited to adjacent jobs (i.e. from job j to job j-1 or job j+1).

The intensive margin elasticity is defined as:

(2.3) $$\xi_j = \frac{c_j - c_{j-1}}{\pi_j} \frac{\partial \pi_j}{\partial \left(c_j - c_{j-1}\right)} .$$

Then it turns out that the optimal taxes satisfy:

(2.4) $$\frac{T_j - T_{j-1}}{c_j - c_{j-1}} = \frac{1}{\xi_j} \frac{\sum_{k=j}^{J} \pi_k \left[1 - g_k - \eta_k \frac{T_k - T_0}{c_k - c_0} \right]}{\pi_j}$$

where g_k is the marginal social value (relative to the value of public funds) of income at job k.

It must be noted that in the model there are no income effects and choices at the intensive margin are restricted in a very special way. Despite these limitations the model is attractive for several reasons:

- it assigns a crucial and easily interpretable role to the two type of elasticities;
- it is simple to implement empirically;
- it seems to fit well into the popular framework that models labour supply as a discrete choice;
- differently from Mirrlees (1971), it allows for negative marginal tax rates (i.e. $T_j < T_{j-1}$): this may be the case if the participation elasticities are sufficiently large with respect to the intensive margin elasticities.

Appendix 3

In Aaberge and Colombino (2006, 2013) the optimal taxation problem is formulated as follows.

$$
\begin{aligned}
&\max_{\vartheta} W\left(U_1\left(c_1,h_1,j_1\right),U_2\left(c_2,h_2,j_2\right),\ldots,U_N\left(c_N,h_N,j_N\right)\right)\\
&\text{s.t.}\\
&\left(c_n,h_n,j_n\right)=\underset{(w,h,j)\in B_n}{\arg\max}\,U_n\left(c,h,j\right) \text{ s.t. } c=f(wh,I_n;\vartheta),\forall n \qquad (3.1)\\
&\sum_{n=1}^{N}\left(w_nh_n+I_n-f(w_nh_n,I_n;\vartheta)\right)\geq R.
\end{aligned}
$$

Agent n can choose a "job" within an opportunity set B_n. Each job is defined by a wage rate w, hours of work h and other characteristics j (unobserved by the analyst). Given gross earnings wh and gross unearned income I, net available income is determined by a tax-transfer function $c=f(wh,I;\vartheta)$ defined up to a vector of parameters ϑ.

For any given tax-transfer rule (i.e. any given value of ϑ) the choices by the agents are simulated by a microeconometric model that allows for a very flexible representation of heterogeneous preferences and opportunity sets, it covers both singles and couples, accounts for quantity constraints and is able to treat any tax-transfer rule however complex. Note that it would be hopeless to look for analytical solutions of an optimal taxation problem in such an environment.

The choices made by the N agents result in N positions $\left(c_1,h_1,j_1\right),\left(c_2,h_2,j_2\right),\ldots,\left(c_N,h_N,j_N\right)$ which are then evaluated by the social planner according to a Social Welfare function W. The Social Planner's problem therefore consists of searching for the value of the parameters ϑ that maximizes W subject to the following constraints:

(1) the various positions $\left(c_1,h_1,j_1\right),\ldots,\left(c_N,h_N,j_N\right)$ result from utility-maximizing choices on the part of the agents (incentive-compatibility constraints);
(2) total net tax revenue must attain a given amount R (public budget constraint).

Problem (3.1) is solved computationally by iteratively simulating the household choices for different values of ϑ until W is maximized.

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