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COMPENSATED LABOR SUPPLY PROBABILITIES AND SLUTSKY ELASTICITIES IN DISCRETE LABOR SUPPLY MODELS

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Compensated Labor Supply Probabilities and Slutsky Elasticities in Discrete Labor Supply Models[1]

by

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Abstract

The methodology of Compensating Variation and Compensated Choice Probability was developed recently by Dagsvik and Karlström (2005). Below we demonstrate how one can apply this methodology in practice. In particular, we compute compensated labor supply probabilities and compensated wage elasticities in a particular discrete labor supply model. The results clearly indicate that the uncompensated and the compensated wage elasticities vary considerably between agents. Thus, heterogeneity seems to be an important issue in labor supply. The uncompensated wage elasticities are higher at the intensive margin than at the extensive margin, while the opposite tend to be the case for the compensated elasticities. Both elasticities tend to decline with the wage level. In standard microeconometric models with deterministic preferences the Slutsky equation implies that if the non-labor elasticity is negative, then the compensated wage elasticity is higher than the uncompensated. With random utility models the Slutsky equation does not exist and we demonstrate empirically that in a majority of cases a negative non-labor elasticity does not imply that the compensated wage elasticity is the highest. Moreover we show that compensated elasticities of hours supplied are substantially lower in random utility models than in traditional models with deterministic preferences.

JEL classification: J22, C51

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1. Introduction

Recently, labor supply models based on the theory of discrete choice and random utility formulations have become increasingly popular. The main reason is that they are much more practical than the conventional continuous approach based on marginal calculus, see the surveys by Creedy and Kalb (2005) and Dagsvik et al. (2012). The discrete approach differs from the corresponding continuous one in that the set of feasible hours of work is approximated by a suitable and finite discrete choice set. There are basically two versions of discrete models of labor supply that have been proposed in the literature. Van Soest (1995) and van Soest et al. (2002) proposed to analyze labor supply as a standard discrete choice problem. In contrast, Dagsvik et al. (1995), with further extension by Dagsvik and Strøm (2006), proposed to analyze labor supply as a job choice problem, where the set of feasible jobs is individual specific and latent.

With the discrete choice approach, it is easy to deal with nonlinear and non-convex economic budget constraints, and to apply rather general functional forms of the utility representations. Whereas it is fairly straight forward to compute uncompensated responses and elasticities in discrete choice models, the computation of the corresponding compensated effects is not trivial.

Dagsvik and Karlström (2005) have demonstrated how one can calculate Compensating Variation (CV) and compensated choice probabilities (Hicks choice probabilities) in random utility models, in the general case when the deterministic part of the utility function may be nonlinear in income. Dagsvik et al. (2009) applied the methodology of Dagsvik and Karlström (2005) to compute welfare measures such as CV in the context of selected tax reforms.

In this paper we show how one can apply the methodology of Dagsvik and Karlström (2005) to compute Hicks choice probabilities and corresponding compensated wage elasticities (Slutsky elasticities). To this end we base our analysis on the model proposed and estimated by Dagsvik and Strøm (2006). Specifically, we show how one can compute Hicks joint choice probabilities of being in particular states before and after wage rate changes or tax reforms. The states are non-working, working in specific sectors and with different working loads. Slutsky elasticities are derived from the Hicks joint probabilities.

A less known, at least less cited paper, in consumer behavior, is Quandt (1956). He confronts the established wisdom at the time by arguing that it might be better to define preferences and indifference in a probabilistic sense and that by doing so the prevailing notion of rationality has to be eliminated. In his model the consumer's action has both a systematic and a random component. Like in Dagsvik and Karlstrøm (2005) he introduces the notion of iso-probability curves, and instead of indifference curves he operates with indifference bands[5]: "„,The consumer is often ignorant of the exact state of his preferences and he is frequently insensitive to small changes or differences in stimuli. As a result, a small movement in any direction from any initial position may leave the consumer as well off as before. It might be suggested that we deal with this problem by considering an indifference map consisting not of indifference curves but of indifference bands„,". Our paper is very much in line with this way of thinking about consumer behavior. Quandt did not do any calculation of the sort presented below.

First we employ the model to calculate the impact of hypothetical changes in wage rates on labor supply choices. We report uncompensated as well compensated wage elasticities, and also non-labor income elasticities. These calculations show that the compensated wage elasticities (Slutsky elasticities) tend to be higher at the extensive margin than at the intensive margin, while the oppsite is true for uncompensated wage elasticities. The elasticities vary considerably with the wage level (the higher the wage is, the lower is the elasticity) and with household characteristics. Thus in labor supply, heterogeneity seems to be an important issue. Moreover, the calculation of the three elasticities demonstrates that the Slutsky equation when preferences are deterministic is not valid within random utility models.

Second, we calculate Slutsky elasticities based on the sample values that were used in estimating the model and compare them with elasticities based on an approach where labor supply is derived from deterministic preferences. The RUM elasticities of conditional expected hours, given working, are around one third of the deterministic ones. The reason is what Quandt pointed to: when indifference curves are replaced by indifference bands, responses to economic incentives become weaker.

The paper is organized as follows. In the next section we review some basic definitions and formulas given by Dagsvik and Karlström (2005) for computing compensated choice probabilities. Subsequently, we discuss special cases such as the binary and the ternary choice setting. This is done for the sake of bringing out the essentials of the methodology of Dagsvik and Karlström (2005), also when it is applied to labor supply. Section 3 gives a brief description of the empirical labor supply model that is the main focus of our application and reports numerical results. Section 4 concludes.

[5] In Appendix D we give an illustration of indifference bands.

2. Compensated choice in random utility models

The history of random utility models (RUM) dates back at least to Thurstone's (1927) analysis. It took, however, around 30 years before this type of models were introduced in economics. See Quandt (1956), Luce (1959), Marschak (1959), Block and Marschak (1960), and McFadden (1973, 1975, 1976, 1978, 1981) for extensive discussions on motivation, exposition and applications. The starting point of Thurstone's modelling approach was that agents were found to behave inconsistently in repeated choice experiments in the sense that they often selected different alternatives at different points in time under seemingly identical experimental conditions.

In micro economics, the theory of compensated (Hicksian) demand and supply plays an important role. Until recently, there has, however, been very little focus on a corresponding theory in the context of random utility models. To our knowledge, the first systematic analysis of compensated choice in random utility models was undertaken by Dagsvik and Karlström (2005). Here, we shall give a brief account of their approach.

We consider a setting with a finite number of alternatives $\{1,2,...,m\}$. Assume that the utility of alternative *j* has the structure $U_j(y)=v_j(y)+\varepsilon_j$, where $v_j(y)$ is a deterministic and monotonically increasing function of y and it may also depend on prices, non-pecuniary and alternative *j* - specific attributes, whereas ε_j is a stochastic term that is supposed to account for the effect on preferences from variables that are not observed by the researcher. For our purpose we only need to introduce income in the notation. In the context of this paper we consider situations where a reform is introduced, and it is therefore convenient to introduce additional notation. To this end let $U_j^0=U_j(y^0)=v_j^0(y^0)+\varepsilon_j$ denote the ex ante utility of alternative *j*, where y^0 denotes the initial income and $v_j^0(\cdot)$ is the deterministic term associated with the utility of alternative *j* ex ante, and let $U_j(y)=v_j(y)+\varepsilon_j$ be the corresponding utility of alternative *j* ex post. Here it is assumed that the stochastic terms of the utility function are not affected by the reform. Let $P^H(j,k)$ denote the joint compensated (Hicksian) probability of choosing alternative *j* ex ante and alternative *k* ex post under the condition that the respective utility levels of the chosen alternatives before and after the reform are the same. In other words, (2.1) $$P^H(j,k) = P(U_j^0 = \max_r U_r^0, U_k(Y) = \max_r U_r(Y), \max_r U_r^0 = \max_r U_r(Y)),$$

where Y is the income required to maintain the utility level equal to the original utility level. Note that the income Y is stochastic due to the utility function containing a stochastic term. Assume in the following that the random error terms are iid with c.d.f. $\exp(-\exp(-x))$. Let y_j be determined by $v_j^0(y^0) = v_j(y_j)$. That is, y_j is the ex post income that ensures that the ex ante and ex post utility of alternative *j* are equal. For *j* to be the most preferred alternative ex ante and *k* the most preferred alternative ex post this implies that $v_k(Y) + \varepsilon_k = v_j^0(y^0) + \varepsilon_j > v_k^0(y^0) + \varepsilon_k$, which implies that $Y \geq y_k$. Furthermore, since alternative k is the most preferred one ex post, $v_k(Y) + \varepsilon_k > v_j(Y) + \varepsilon_j$, which together with the above result yields $v_j(Y) + \varepsilon_j < v_j^0(y^0) + \varepsilon_j$. The latter inequality implies that $Y \leq y_j$. Hence, for transitions from alternative *j* to alternative *k* to take place, under constant indirect utility level it must be the case that $y_k \leq Y \leq y_j$. From Dagsvik and Karlström (2005), page 67 and Corollary 3, it follows that

(2.2) $$P^H(j,k) = \int_{y_k}^{y_j} \frac{\exp(v_j^0(y^0))\exp(v_k(y))dv_k(y)}{\left\{\sum_{r=1}^m \exp(\psi_r(y))\right\}^2} = \exp(v_j^0(y^0)) \int_{y_k}^{y_j} \frac{\exp(v_k(y))dv_k(y)}{\left\{\sum_{r=1}^m \exp(\psi_r(y))\right\}^2},$$

where $\psi_r(y) = \max(v_r^0(y^0), v_r(y))$. In the case where *j* and *k* are distinct and $y_j \leq y_k$, then $P^H(j,k) = 0$. Furthermore, when $U_j^0 = \max_r(U_r^0, U_r(y_j))$, then $Y = y_j$ and the ex post choice will be equal to the ex ante choice. The corresponding probability is equal to

(2.3) $$P^H(j,j) = P(U_j^0 = \max_r(U_r^0, U_r(y_j)) = \frac{\exp(v_j^0(y^0))}{\sum_{r=1}^m \exp(\psi_r(y_j))} = \frac{\exp(v_j^0(y^0))}{\sum_{r=1, r\neq j}^m \exp(\psi_r(y_j)) + \exp(v_j^0(y^0)}.$$

2.1. The binary case

We shall now consider the binary case in more detail. From (2.2) we obtain that

(2.4) $$P^H(1,2)=\int_{y_2}^{y_1}\frac{\exp(v_1^0(y^0))\exp(v_2(y))dv_2(y)}{\left\{\sum_{k=1}^{2}\exp(\psi_k(y))\right\}^2}=\exp(v_1^0(y^0))\int_{y_2}^{y_1}\frac{\exp(v_2(y))dv_2(y)}{\left\{\sum_{k=1}^{2}\exp(\psi_k(y))\right\}^2}\ .$$

Integration yields

(2.5) $$P^H(1,2)=-\exp(v_1^0(y^0))\ \Big|_{y_2}^{y_1}\frac{1}{\exp(v_1^0(y^0))+\exp(v_2(y))}.$$

$$=\frac{\exp(v_1^0(y^0))}{\exp(v_1^0(y^0))+\exp(v_2^0(y^0))}-\frac{\exp(v_1^0(y^0))}{\exp(v_1^0(y^0))+\exp(v_2(y_1))}$$

The probability $P^H(2,2)$ is given by

(2.6) $$P^H(2,2)=\frac{\exp(v_2^0)}{\sum_{k=1}^{2}\exp(\psi_k(y_2))}=\frac{\exp(v_2^0)}{\exp(v_1^0)+\exp(v_2^0)}.$$

Consequently, it follows that

(2.7) $$P_2^H=P^H(1,2)+P^H(2,2)=\frac{\exp v_2(y_1)}{\exp(v_1^0)+\exp(v_2(y_1))}.$$

The corresponding ex ante probability of choosing alternative 2, is given by

(2.8) $$P_2=\frac{\exp(v_2^0(y^0))}{\exp(v_1^0(y^0))+\exp(v_2^0(y^0))}.$$

It follows that the compensated effect, when measured by the relative change $(P_2^H-P_2)/P_2$, is given by

(2.9) $$\frac{P_2^H-P_2}{P_2}=\left(\frac{\exp(v_2(y_1))}{\exp(v_1^0(y^0))+\exp(v_2(y_1))}-\frac{\exp(v_2^0(y^0))}{\exp(v_1^0(y^0))+\exp(v_2^0(y^0))}\right)/P_2$$

$$=\frac{\exp(v_2(y_1))-\exp(v_2^0(y^0))}{\exp(v_1^0(y^0))+\exp(v_2(y_1))}=\frac{\exp(v_2(y_1))-\exp(v_2(y_2))}{\exp(v_1^0(y^0))+\exp(v_2(y_1))}>0,$$

because $y_1 > y_2$.

In the case where $y_1 \le y_2$, $P^H(1,2)=0$, and $P_2^H=P^H(2,2)$, so that we now obtain

(2.10) $$P_2^H = P^H(2,2) = \frac{\exp(v_2^0(y^0))}{\exp(v_2^0(y^0)) + \exp(v_1(y_2))}.$$

Hence, in this case

(2.11) $$\frac{P_2^H - P_2}{P_2} = \frac{\exp(v_1^0(y^0)) - \exp(v_1(y_2))}{\exp(v_2^0(y^0)) + \exp(v_1(y_2))} = \frac{\exp(v_1(y_1)) - \exp(v_1(y_2))}{\exp(v_2^0(y^0)) + \exp(v_1(y_2))} \le 0,$$

since $y_1 \le y_2$.

Consider next the special case with only a change in the price (or cost) whereas other attributes (or tax rules remain fixed). In this case $v_1(y) = v_1^0(y),$ so that $y_1 = y^0.$ Since $v_2(y) < v_2^0(y)$ for any y, and y_2 is determined by $v_2(y_2) = v_2^0(y^0),$ it must be the case that $y_2 < y^0 = y_1.$ According to the analysis above, this means that the compensated (Slutsky) price elasticity can never be positive.

2.2. The ternary case

Consider finally the case with 3 alternatives, i.e., $m = 3$. Then (2.2) reduces to

(2.12) $$P^H(j,k) == \exp(v_j^0(y^0)) \int_{y_k}^{y_j} \frac{\exp(v_k(y))dv_k(y)}{\left\{\sum_{r=1}^{3} \exp(\psi_r(y))\right\}^2},$$

for $j, k = 1,2,3$, and distinct j and k. Suppose for example that $y_1 > y_3 > y_2.$ Then it follows that $P^H(1,2) > 0,$ $P^H(1,3) > 0,$ $P^H(3,2) > 0,$ whereas $P^H(2,3) = P^H(3,1) = P^H(2,1) = 0.$ We obtain that

(2.13) $$P^H(3,2) = \exp(v_3^0(y^0)) \int_{y_3}^{y_1} \frac{\exp(v_2(y))dv_k(y)}{\left\{\exp(v_1^0(y^0)) + \exp(v_2(y)) + \exp(v_3^0(y^0))\right\}^2}$$

$$= \frac{\exp(v_3^0(y^0))}{\exp(v_1^0(y^0)) + \exp(v_2(y_3)) + \exp(v_3^0(y^0))} - \frac{\exp(v_3^0(y^0))}{\exp(v_1^0(y^0)) + \exp(v_2(y_1)) + \exp(v_3^0(y^0))}$$

$$= \frac{\exp(v_3^0(y^0))(\exp(v_2(y_1)) - \exp(v_2(y_1))}{(\exp(v_1^0(y^0)) + \exp(v_2(y_3)) + \exp(v_3^0(y^0)))(\exp(v_1^0(y^0)) + \exp(v_2(y_1)) + \exp(v_3^0(y^0)))}.$$

However, for $P^H(1,2)$ one cannot in general express the integral on closed form. The ex post Hicksian probability of choosing alternative 2 equals

(2.14) $$P_2^H = P^H(1,2) + P^H(3,2) + P^H(2,2),$$

and similarly for the other cases. Since $P^H(3,1) = P^H(2,1) = 0$, we get for alternative 1 that

$$P_1^H = P^H(1,1) = \frac{\exp(v_1^0(y^0))}{\exp(v_1^0(y^0)) + \exp(v_2(y_1)) + \exp(v_3(y_1))}. \tag{2.15}$$

2.3. Labor market examples

Consider now the case of a model for labor force participation (binary case, j=1,2) and subsequently a model for labor force participation and choice of working in one out of two sectors (ternary case, j=1,2,3).

Example 1: Labor force participation (the binary case)

Consider first the choice of whether or not to work. Let w be the agent's wage, I the agent's non-labor income and let full time hours of work be normalized to one. Let $f(hw, I)$ denote income after tax, where h = 1 if working and h = 0, otherwise, and let $u(C,h)$ denote the mean utility of disposable income and hours of work (C, h). $U_1 = u(f(0,I),0) + \varepsilon_1$, be the agent's utility of not working and $U_2 = u(f(w,I),1) + \varepsilon_2$, the utility of working, were the random error term ε_j, j = 1, 2, are supposed to account for unobserved heterogeneity in preferences across alternatives and agents. The agent will work if $U_2 > U_1$. In this example it is for simplicity assumed that the agent has no problem with finding a job, and consequently the event of being employed will therefore be determined by the agent's preferences.

Consider now the effect of a change in economic incentives (such as a wage increase or a tax reform) that makes the working alternative more (or less) attractive. We assume that the random part of the utility function is unaffected by the change in taxation. Let w^0, f^0 be the ex ante wage and tax system, and w^1, f^1, the corresponding wage and tax system ex post. Moreover, let Y be the non-labor income that makes the ex post indirect utility equal to the ex ante indirect utility. For simplicity, write $v_j^0(I) = u(f^0((j-1)w^0, I), j-1)$, $U_j^0 = v_j^0(I) + \varepsilon_j$, $v_j(Y) = u(f((j-1)w, Y), j-1)$, and $U_j(Y) = v_j(Y) + \varepsilon_j$, for j = 1, 2. Thus U_j^0 is the ex ante utility of being in labor market state j and $U_j(Y)$ the corresponding ex post utility. The random parts ε_j, j = 1, 2, are assumed iid extreme value distributed across alternatives. The deterministic utilities may also depend on other alternative specific attributes. Let y_j be defined by $v_j^0(I) = v_j(y_j)$, j = 1, 2. The income y_j is a money metric measure of the initial deterministic utilities. The compensated probability of “working” (“not working”) can now be analyzed as outlined about in section 2.1. In particular, it follows that the compensated wage elasticity (Slutsky) will never be non-positive.

Example 2: A two sector labor supply model (the ternary case)

Consider finally a particular two sector labor supply model. In this case there are 3 alternatives, “not working” (1), “working in the public sector” (2) and “working in the private sector” (3). Here, and for expository reasons, in each sector the only option is to work full time. As above let U_1 be the utility of not working, U_2 the utility of working in sector 2 and U_3 the utility of working in sector 2. Assume that $U_j(I) = u_j(f(w_j, I), 1) + \varepsilon_j$ for j = 2, 3, where w_j is the wage of sector *j*. Note that the function u_j may depend on j because the systematic part of the utility function may depend on attributes of the sectors other than wage. Let $v_1(I) = u(f(0, I), 0) + \varepsilon_1$ and $v_j(I) = u_j(f(w_j, I), 1)$, for *j* = 2, 3, and $U_j(I) = v_j(I) + \varepsilon_j$. Similar to the analysis in the previous example, let $U_j^0 = v_j^0(I) + \varepsilon_j$ be the ex ante utility representation and $U_j(y) = v_j(y) + \varepsilon_j$ the corresponding ex post utility representation. The analysis now proceeds similarly to the ternary case discussed above.

3. Analysis of compensated choice in a particular discrete labor supply model

The labor supply model that will be employed in the calculation of compensated choice probabilities and Slutsky elasticities is based on Dagsvik and Strøm (2006), Dagsvik et al. (2009) and Locatelli and Strøm (2011). Here we only give a brief review of the model. First, only female labor supply is modeled. The wage income of the husband is assumed exogenously given. Second, the female can choose between not working and working different hours of work in the private and the public sector. In each sector the female can choose between the following hours, h: {315, 780, 1040, 1560, 1976, 2340, 2600}. The household derives utility from household consumption, here set equal to household disposable income, leisure and non pecuniary latent attributes of jobs.

For expository simplicity we shall however only consider the one sector model in this section. An outline of the general two sector model is given in Appendix A. Summary statistics, tax functions and estimates are given in Appendix B. Let z = 1, 2,,, be an indexation of the jobs and let z = 0 represent not working. The utility function is assumed to have the form $U(C,h,z)=\log v(C,h)+\varepsilon(z)$, for $z\in B$ where B is the set of available jobs, (C, h) denotes disposable income and annual hours of work, $v(\cdot)$ is a suitable positive deterministic function. The set B is individual specific and latent. The terms $\{\varepsilon(z)\}$ are positive sector-and job-specific random taste shifters. The taste shifter accounts for unobserved individual characteristics and unobserved job-specific attributes. These taste shifters $\{\varepsilon(z)\}$, are assumed to be i.i.d. across jobs and agents, with c.d.f. $\exp(-\exp(-x))$, for real x. The reason why the index z enters the utility function is that job-specific attributes beyond wage and hours of work may affect the utility of the agents. For given hours of work h and wage rate w, disposable household income is given by $C=f(hw,I)$, where $f(\cdot)$ is a function that transforms pre-tax incomes into after-tax incomes, w is the woman's wage and I denotes non-labor income. It is convenient to decompose the set B as the union of the sets $\{B(h)\}$, where $B(h)$ is the set of available jobs with hours of work equal to h. Let θ be a measure that represents the number of available jobs and $g(h)$ the fraction of jobs with hours of work that are feasible. Let $\varphi(h\mid w,I)$ be the probability (uncompensated) of choosing hours of work h (for a utility maximizing agent), and let D be the set of feasible hours. From the assumptions above it follows readily that

$$\varphi(h)=\frac{v\big(f(hw,I),h\big)g(h)\theta}{v(f(0,I),0)+\sum_{x>0,x\in D}v\big(f(xw,I),x\big)g(x)\theta} \tag{3.1}$$

for $h>0$, see Dagsvik and Strøm (2006). For $h = 0$, $\varphi(0\mid w,I)$ is obtained from (3.1) by replacing the numerator by $v(f(0, I), 0)$.

Similarly to Section 2, we now consider a setting where a tax reform, a wage change or some other change is introduced. For example, the framework above allows for changes in latent choice constraints through the opportunity measure $\theta g(h)$. the corresponding compensated effects can now be calculated in a similar way as outlined in Section 2. To this end, let

$$V^0(h,y)=\max_{z\in B(h)}(\log v^0(f^0(hw^0,y),h)+\varepsilon(z))$$

where the zero superscript denotes the initial ex ante situation. Thus $V^0(h,y)$ is the ex ante utility of working h hours when non-labor income is equal to y. Similarly, the corresponding ex post utility of working h hours can be expressed as

$$V(h,y)=\max_{z\in B(h)}(\log v(f(hw,y),h)+\varepsilon(z)).$$

Let $y(h)$ be the real number that solves:

$v^0(f^0(hw^0,I),h)+\log(\theta^0 g^0(h)) = v(f(hw,y(h)),h)+\log(\theta g(h)),$ when h is positive and

$v^0(f^0(0,I),0) = v(f(0,y(0)),0),$ when h = 0. Furthermore, let $\bar{D} = D \cup \{0\}.$ Define the joint compensated probability

(3.2) $P^H(h,\tilde{h})$

$$= P(\max_{x\in\bar{D}} V^0(x,I) = V^0(h,I), V(\tilde{h},Y) = \max_{x\in\bar{D}} V(x,Y), \max_{x\in\bar{D}} V^0(x,I) = \max_{x\in\bar{D}} V(x,Y)\}.$$

The probability defined in (3.2) is entirely similar to the corresponding one defined in section 2. It is the probability that the ex ante labor supply is equal to h and the ex post supply is equal to $\tilde{h}$, when the ex ante and ex post utility levels are equal. It follows from Dagsvik and Karlström (2005) that

$$(3.3)\qquad P^H(h,\tilde{h}) = 1\{y(h) > y(\tilde{h})\}\int_{y(\tilde{h})}^{y(h)} \frac{\theta^0 g^0(h)\theta g(\tilde{h}) v^0(f^0(hw^0,I),h) v(f(\tilde{h}w,dy),\tilde{h})}{K(y)^2},$$

for $h \neq \tilde{h},$ and $h > 0,$ $\tilde{h} > 0,$ where the indicator notation $1\{\cdot\}$ means that $1\{y > x\}$ if $y > x$ and zero otherwise and

$$(3.4)\quad K(y) = \max(v^0(f^0(0,I),0), v(f(0,y),0)) + \sum_{x\in D} \max[\theta^0 g^0(x) v^0(f^0(xw^0,I),h), \theta g(x) v(f(xw,y),x)].$$

Note that (3.3) allows for changes in the opportunity measure, where $\theta^0 g^0(h)$ and $\theta g(h)$ denote the ex ante and ex post opportunity measures, respectively. Furthermore, we get, similarly to (2.3) that

$$(3.5)\qquad P^H(h,h) = \frac{\theta^0 g^0(h) v^0(f^0(hw^0,I),h))}{K(y(h))},$$

for $h > 0$. For $h = 0$, $\tilde{h} > 0$ it follows that

$$(3.6)\qquad P^H(0,\tilde{h}) = 1\{y(0) > y(\tilde{h})\}\int_{y(\tilde{h})}^{y(0)} \frac{\theta g(\tilde{h}) v^0(f^0(0,I),0) v(f(\tilde{h}w,dy),\tilde{h})}{K(y)^2},$$

and similarly for $h > 0, \tilde{h} = 0.$ For $h = \tilde{h} = 0,$ we have that

(3.7) $$P^H(0,0) = \frac{v^0(f^0(0,I),0))}{K(y(0))}.$$

Let $\varphi^0(h)$ denote the ex ante choice probability, which is obtained from (3.1) by inserting the initial tax system, wage and opportunity distribution. The marginal ex post compensated choice probability, $\varphi^H(h)$ is given by

(3.8) $$\varphi^H(h) = \sum_{x \in \bar{D}} P(x,h).$$

Hence, the compensated relative change in the probability of choosing h hours of work induced by the reform equals

(3.9) $$\frac{\varphi^H(h) - \varphi^0(h)}{\varphi^0(h)}.$$

The corresponding compensated change in the mean labor supply is given by

(3.10) $$\frac{\sum_{x \in D} x\varphi^H(x) - \sum_{x \in D} x\varphi^0(x)}{\sum_{x \in D} x\varphi^0(x)}.$$

3.1. Elasticities generated by hypothetical changes in wage rates

In two important papers Haavelmo (1943;1944) formalized the distinction between correlation and causation. According to Haavelmo causal effects say, of variation in wages on hours supplied, are defined using a hypothetical model that abstracts from the empirical data generating process by making hypothetical changes in wages. We have applied this thought experiment here and used our model, once it is estimated, to calculate the impact of hypothetical changes in wages and non-labor income on labor supply.

In Appendix C we report uncompensated and compensated wage elasticities of labor supply (extensive and intensive margin) as well as non-labor income elasticities for 48 different cases. We have chosen three wage levels (NOK 1994), which is low (NOK 70 per hour), high (NOK 200 per hour) and super-high (NOK 300 per hour). Non-labor incomes (which includes the income of the husband) are NOK 50 000 (low), NOK 100 000 (lower than average), NOK 200 000 (around average) and NOK 400 000 (high). Household characteristics are no children or 2, and the age of the woman is either 30 or 40. The results shown below clearly indicate that heterogeneity is an important issue in labor supply. Moreover, in random utility model the Slutsky equation similar to in models with deterministic preferences does not exist.

Heterogeneity

Table 1 shows that there is large variation in the uncompensated elasticities. The highest elasticity of conditional hours with respect to the wage is around 4 (public sector) and 2.5 (the private sector) times higher than the lowest elasticity. For the elasticity of unconditional hours the ratios are around 9 (public sector) and 6 (private sector).

The differences between the highest and lowest compensated elasticities are smaller (Table 2) than for the uncompensated.

Table 3 shows that the non-labor income elasticities can also be positive. Moreover the difference between the lowest and the highest elasticities is sizeable.

Table 1. Uncompensated wage elasticities

High: Woman aged 40, two children, non-labor income NOK 400 000, wage level NOK70
Low: Woman aged 30, no children, non-labor income NOK 50 000, wage level NOK 70

Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
	All	Public	Private	All	Public	Private	All	Public	Private
High	0.724	0.763	0.606	0.589	0.602	0.537	1.356	1.411	1.117
Low	0.000	0.007	-0.043	0.163	0.152	0.222	0.163	0.160	0.178

Table 2. Compensated wage elasticities

High: Woman aged 40, two children, non-labor income NOK 50 000, wage level NOK 300
Low: Woman aged 30, no children, non-labor income NOK 50 000, wage level NOK 70

Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
	All	Public	Private	All	Public	Private	All	Public	Private
High	0.419	0.331	0.765	0.267	0.259	0.292	0.687	0.590	1.057
Low	0.131	0.135	0.119	0.186	0.174	0.254	0.318	0.310	0.368

Table 3. Non-labor income elasticities

High: Woman aged 30, no children, non-labor income NOK 400 000, wage level NOK 200
Low: Woman aged 30, two children, non-labor income NOK 100 000, wage level NOK 70

Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
	All	Public	Private	All	Public	Private	All	Public	Private
High	0.000	-0.003	0.018	0.005	0.003	0.004	0.005	0.001	0.023
Low	-0.126	-0.137	-0.075	-0.149	-0.148	-0.146	-0.273	-0.284	-0.221

Extensive versus intensive margins

The uncompensated wage elasticities is higher at the intensive margin than at the extensive margin, with only two exceptions (out of 48 cases). The compensated wage elasticities are more equal at the extensive and the intensive margin, but with a weak tendency to be higher at the extensive margin. The non-labor income elasticities tend to be lowest (more negative) at the intensive margin.

The variation of wage elasticities with respect to the wage level

The elasticities tend to decline with the wage, in particular for women with two rather no children. For example, for a woman aged 30 with two children and non-labor income NOK 200 000, the uncompensated elasticity of conditional hours (all sectors) declines from 0.634 (wage level NOK70) to 0.244 (wage level NOK 300) and the uncompensated elasticity of unconditional hours from 1.177 to 0.344.

For the same woman and for the same wage levels, the decline in compensated elasticity is smaller: The compensated elasticity of conditional hours declines from 0.410 to 0.266, and the compensated elasticity of unconditional hours drops from 1.093 to 0.635.

The Slutsky equation

In deterministic models and where labor supply is derived from maximizing utility given a budget constraint, the Slutsky equation is

$$E:uncompensated = E:compensated + \frac{I}{mh} E:non-labor$$

where m is marginal wage rate and h is optimal hours. As above I is non-labor income. We observe that a negative elasticity of hours with respect to non-labor income implies that the uncompensated elasticity of hours with respect to the wage is lower than the compensated. Almost in half of the cases shown in Appendix C the opposite is true for the elasticities of conditional hours with respect to the wage. Table 4 gives two examples. In both cases the non-labor income elasticity is negative. In one case the uncompensated wage elasticity is higher than the compensated, in the other case the opposite is true. In both cases the woman is 40 years old, she has two children and her wage rate is NOK 70. The only difference between the two cases is that the uncompensated elasticity is higher when the non-labor income is high.

Table 4. Uncompensated (M), compensated (S) and non-labor income elasticities (I)

1. **Woman aged 40, two children, wage level NOK 70, non-labor income NOK 50 000**

Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
	All	Public	Private	All	Public	Private	All	Public	Private
M	0.065	0.131	-0.153	0.508	0.500	0.521	0.578	0.638	0.360
S	0.371	0.474	0.011	0.566	0.557	0.571	0.938	1.031	0.582
I	-0.065	-0.082	-0.010	-0.145	-0.143	-0.146	-0.210	-0.224	-0.156

2. **Woman aged 40, two children, wage level NOK 70, non-labor income NOK 400 000**

Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
	All	Public	Private	All	Public	Private	All	Public	Private
M	0.724	0.763	0.606	0.589	0.602	0.537	1.356	1.411	1.117
S	0.066	0.094	-0.019	0.414	0.422	0.378	0.480	0.517	0.358
I	-0.130	-0.129	-0.132	-0.021	-0.021	-0.023	-0.151	-0.150	-0.155

Economic incentives

Because hours are less constrained in the private sector than in the public sector the incentives to start working in the private sector is stronger for a woman with a potential high wage. Hence, for those women the participation elasticities are higher related to working in the private sector than in the public sector.

The elasticities of conditional hours tend to be higher when the wage is low compared to when the wage is high. The reason is that when the wage is high the woman works long hours and hence the impact of a higher wage is lower compared to the case where the wage rate is low and initial hours are lower. This is particular the case given that the woman works in the private sector, where hours are less constrained.

If the woman has children the elasticities of conditional hours is higher compared to a childless woman. The reason is that the presence of small children reduces hours of work and hence the impact of wage increase becomes stronger compared to a case for a childless women who works initially longer hours.

3.2. Numerical results using sample values

This Section contains numerical results for compensated wage elasticities for the two-sector model using sample values. This allows us to show how the compensated elasticities vary across deciles of the (endogenous) income distribution.

The income deciles limits are calculated from the expected household disposable income, using the probabilities in (A.1, Appendix A). For each possible hour we have calculated the mean of the probabilities based on 50x50 draws from the normal distribution to account for the fact that the wage equations contain random terms. Summing over all possible hours and sectors, including not working, we are then able to identify the expected income, inclusive of the exogenous incomes, and hence the income decile limits. Within each decile we then use (eqs. A.4- A.14) to calculate the relative change in compensated probabilities for each hours and sector and finally we calculate weighted average over hours, given the sector, using the compensated probabilities as weights. Again, we have to make draws from the distribution of the error terms in the wage equation since these enter in (eqs.A.4-A.14). To calculate the elasticity of working, we take the weighted average over all deciles and sectors. To calculate the elasticity of working in a specific sector we do an equivalent calculation.

In Tables 5-7 we give the compensated elasticities related to an overall (10%) wage increase. Table 5 gives the elasticities of working, and working in the two sectors, Table 6 gives the elasticities of conditional expected hours, conditional on working and working in specific sectors, while Table 7 gives the elasticities of the unconditional expected hours (which is the sum of the two above).

Table 5. Compensated elasticities of the probability of working, and of working in the public or private sector, across deciles in the household income distribution

Sector	1^{st} decile	$2\text{-}9^{th}$ decile	10^{th} decile
All sectors	0.4170	0.4650	0.4752
Public	0.2840	0.2670	0.2495
Private	0.5232	0.6609	0.8180

Table 6. Compensated elasticities of conditional expected hours in the public or private sector, across deciles in the household income distribution

Sector	1^{st} decile	$2\text{-}9^{th}$ decile	10^{th} decile
All sectors	0.3069	0.2963	0.3256
Public	0.3156	0.3052	0.3240
Private	0.3021	0.2903	0.3409

Table 7. Compensated elasticities of unconditional expected hours in the public or private sector, across deciles in the household income distribution

Sector	1^{st} decile	$2\text{-}9^{th}$ decile	10^{th} decile
All sectors	0.7238	0.7613	0.8009
Public	0.6005	0.5772	0.5734
Private	0.8253	0.9512	1.1590

A striking result is that the elasticities of conditional expected hours seem to be nearly the same across deciles and sectors. With the exception of working in the public sector, the compensated elasticities at the extensive margin tend to be higher than at the intensive margin; this is particular the case for the private sector. This is in accordance with the results reported in the previous section.The elasticities indicate that a utility constant overall wage increase shift labor more into the private sector.

We have also computed the elasticities when the opportunity sets regarding jobs imply that all hours of work are equally feasible ("uniform"), as they are in the Hausman type of labor supply models

(Hausman (1979)). This implies that the densities $g_k(h)$ in eq.(3.1) equal 1. In order to keep the number of total available jobs constant in each sector, θ_k are changed to yield this result. The elasticities are given in Tables 8-10, and for the sake of comparison we have included the elasticities from the Tables above ("base case").

Table 8. Compensated elasticities of the probability of working and of working in the public or the private sector.

Sector	1st deciles		2-9th deciles		10th deciles	
	Base case	Uniform	Base case	Uniform	Base case	Uniform
All sectors	0.4170	0.4340	0.4650	0.4669	0.4752	0.4484
Public	0.2840	0.1598	0.2670	0.1359	0.2495	0.1203
Private	0.5232	0.6513	0.6609	0.7964	0.8180	0.9372

Table 9. Compensated elasticities of conditional expected hours

Sector	1st deciles		2-9th deciles		10th deciles	
	Base case	Uniform	Base case	Uniform	Base case	Uniform
All sectors	0.3069	0.3882	0.2963	0.3829	0.3256	0.4278
Public	0.3165	0.4207	0.3052	0.4054	0.3240	0.4230
Private	0.3021	0.3564	0.2903	0.3452	0.3409	0.4156

Table 10. Compensated elasticities of unconditional expected hours

Sector	1st deciles		2-9th deciles		10th deciles	
	Base case	Uniform	Base case	Uniform	Base case	Uniform
All sectors	0.7238	0.8222	0.7613	0.8498	0.8009	0.8762
Public	0.6005	0.5805	0.5722	0.5413	0.5734	0.5432
Private	0.8053	1.0076	0.9512	1.1416	1.1590	1.3528

The utility constant elasticities related to working at all are almost the same in these two cases and they are almost the same across deciles. Compared to the base case, the case with uniform hours in the choice sets has lower Slutsky elasticities of working in the public sector. Thus, with uniform hours instead of the base case an overall wage increase shifts more labor to the private sector. As expected, the elasticities of conditional hours, conditional on working and working in specific sectors, are higher when the constraints on offered hours in the choice sets are removed ("uniform").

3.3 Compensated wage elasticities when the stochastic part of the utility function is ignored.

In this Section we report compensated wage elasticity of hours, given participation. We use our estimates of the structural part of the model to derive optimal hours. These hours follow from maximizing the structural part of the utility with respect to hours of work, given the budget constraint. Based on this, the compensated elasticities of hours worked with respect to the wage for individual *i*, in sector k = 1,2, S_{ik}, are given by:

$$S_{ik} = \frac{1}{(1-\alpha_1)\dfrac{m_{ki}h_{ki}}{C_{ki}} + (1-\alpha_3)\dfrac{vh_{ki}}{(1-vh_{ki})}}, \tag{3.11}$$

$$m_{ki} = w_{ki} f_1'(h_{ki}w_{ki}, I_i),\ \nu = 1/3640,\ k = 1 \text{ (public)},\ k = 2 \text{ (private)}.$$

In Table 11 we give the values of these Slutsky elasticities across deciles in the household income distribution. The income distribution is calculated by using the choice probabilities in (3.1) to calculate expected income for all individuals in the sample.

Table 11. Mean Slutsky elasticities when utilities are deterministic, given working.

Sectors	First deciles	2-9th deciles	10th deciles
All sectors	0.8164	0.8151	0.9021
Public	0.8857	0.8931	1.0329
Private	0.8676	0.8775	1.0188

We observe that the elasticities are quite similar across sectors and deciles. They tend to be a little higher in the upper deciles.

Comparing Table 10 and Table 6 we observe that when the full randomness of the utility structure is accounted for, relative to when only the deterministic part of the utility function is used, the compensated elasticities are around 1/3 of the elasticities when preferences are deterministic. The reason is that with random utility functions the responses become more sluggish: The substitution does not take place along indifference curves but in indifference bands, as suggested, but not calculated, by Quandt (1956).

4. Conclusion

We have demonstrated how Slutsky elasticities can be calculated in random utility models (RUM) and we also show numerical results based on a model estimated on Norwegian female labor supply data from 1994. We have compared the results when using the whole model with the results when only the deterministic part of the utility function is used. The elasticities of conditional expected hours, conditional on working, are around 1/3 in the RUM compared to in the case with deterministic preferences. As pointed out by Quandt (1956) this is to be expected when preferences are random. With random preferences indifference curves do not exist and has to be replaced by iso-probability curves and indifference bands. Moreover, the individuals are assumed to choose between working or not, working in the public and private sector, and to work different hours of work. Thus, a constant utility wage increase may give rise to different substitutions.

The results in Section 3.1 and in Appendix C clearly indicate that the uncompensated and the compensated wage elasticities vary considerably between agents. Thus, heterogeneity seems to be an important issue in labor supply. The uncompensated wage elasticities is higher at the intensive margin than at the extensive margin, while the opposite tend to be the case for compensated elasticities. Both elasticities tend to decline with the wage level.

In standard microeconometric models with deterministic preferences the Slutsky equation implies that if the non-labor income elasticity is negative, then the compensated wage elasticity is higher than the uncompensated. With random utility models the Slutsky equation does not exist and we demonstrate empirically that in a many cases a negative non-labor elasticity does not imply that the compensated wage elasticity is the highest.

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Appendix A. The two sector discrete labor supply model

This section outlines the two sector job choice model for married females. Here it is assumed that the female takes her husbands income as given. Let w_k denote the wage the female receives when working in sector *k*, *k* = 1, 2, and let $w=(w_1,w_2)$. The budget constraint when working in sector *k, k* = 1, 2, is given by

$$C_k = f(hw_k, I) \equiv hw_k - T(hw_k) + I,$$

where *T* is the tax function, *h* is hours of work and I is the sum of three income components. These three incomes are the after tax wage income of the husband, the capital income (taxed at 28 per cent) of the household and child allowances, which vary with the number of children up to the age of 18. Child allowances are not taxed. All details of the tax structure *T*(.) are taken into account in the estimation and simulation of the model. Let $\varphi_k(h)$ be the uncompensated probability of choosing a job in sector *k* with hours of work *h* (for an utility maximizing agent), and let *D* be the set of feasible hours (assumed to be the same across sectors). Similarly to Section 3, it is demonstrated by Dagsvik and Strøm (2006) that

(A.1) $$\varphi_k(h) = \frac{v\big(f(hw_k, I), h\big) g_k(h)\theta_k}{v\big(f(0,y),0\big) + \sum_{r=1}^{2} \sum_{x>0, x\in D} v\big(f(xw_r, I), x\big) g_r(x)\theta_r}$$

for $h > 0$, $k = 1,2$, where $g_k(h)$ denotes the fraction of jobs with hours of work that are available in sector *k* (available to the agent). The term θ_k is a measure of the total amount of jobs available to the female in sector *k*, *k* =1,2. For *h* = 0, $\varphi(0)$ is obtained from (A.1) by replacing the numerator by $v(f(0, y), 0)$.

The deterministic part of the utility function is specified as a Box-Cox transformation of consumption and leisure:

$$
\text{(A.2)}\quad \log v(C,h)=\alpha_2\left(\frac{\left[10^{-4}(C-C_0)\right]^{\alpha_1}-1}{\alpha_1}\right)+\alpha_9\left(\frac{\left[10^{-4}(C-C_0)\right]^{\alpha_1}-1}{\alpha_1}\right)\left(\frac{(1-h/3640)^{\alpha_3}-1}{\alpha_3}\right)
$$

$$
+(\alpha_4+\alpha_5\log A+\alpha_6(\log A)^2+\alpha_7 X_{0,6}+\alpha_8 X_{7,17})\left(\frac{(1-h/3640)^{\alpha_3}-1}{\alpha_3}\right).
$$

Here C_0 is minimum consumption; A is age and X_{06}, $X_{7,17}$ are the number of children below 6 and between 7 and 17, respectively. The alfa-s are unknown coefficients. If α_1 and α_3 are below 1, the deterministic part of the utility function is strictly concave. In order to make the paper self-contained, information about data, tax functions and estimates is appended (see Tables B.1 -B.4).

Consider now the calculation of the compensated choice probabilities in the context of a reform of the tax system or a change in the wages. Here we shall assume that the opportunity measures $\theta_k g_k(h)$ remain unaffected by the reform. Let f^0 and f represent the initial and ex post tax system and w_k^0 and w_k the initial and ex post wage in sector k.

Let $y_k(h)$ be defined by $v(f^0(hw_k^0,I),h)=v(f(hw,y_k(h)),h),$ for positive h and k, and let $y_0(0)=y(0),$ for $h = k = 0$.The function $y_k(h)$ is the ex post non-labor income that makes the ex ante deterministic part of utility equal to the corresponding ex post part. Since the function v and the opportunity measures are unaffected by the reform it follows that the equation above is equivalent to

$$
\text{(A.3)}\qquad f^0(hw_k^0,I)=f(hw_k,y_k(h)).
$$

Similarly to the definition in (3.2) let $P^H(j,h,k,\tilde{h})$ denote the joint probability of being in sector j working h hours ex ante and in sector k working $\tilde{h}$ hours, given that the ex ante and ex post maximal utilities are the same. Here j, k = 0,1, 2, where by sector 0 we understand the alternative not working. As in (3.3) it follows from Dagsvik and Karlström (2005) that

$$
\text{(A.4)}\quad P^H(j,h,k,\tilde{h})=1\{y_j(h)>y_k(\tilde{h})\}\int_{y_k(\tilde{h})}^{y_j(h)}\frac{g_j(h)g_k(\tilde{h})\theta_j\theta_k v(f^0(hw_j^0,I),h)v(f(\tilde{h}w_k,dy),\tilde{h})}{K(y)^2},
$$

which is valid for positive hours and $k > 0$, $j > 0$, $(j,h)\neq(k,\tilde{h})$. Due to (A.2) it follows that

$$
\begin{aligned}
\text{(A.5)} \qquad K(y) &= \max(v(f^0(0,I),0), v(f(0,y),0)) \\
&+ \sum_{r=1}^{2} \sum_{h \in D} \max(g_r(h)\theta_r v(f^0(hw_r^0, I), h), g_r(h)\theta_r v(f(hw_r, y), h))
\end{aligned}
$$

and

(A.6) $$v(f(hw_k, dy), h) = \left(\alpha_2 + \alpha_9 \left(\frac{(1-h/3640)^{\alpha_3} - 1}{\alpha_3}\right)\right)\left(10^{-4}(f(hw_k, y) - C_o)\right)^{\alpha_1 - 1} v(f(hw_k, y), h)dy.$$

The cases where one or two alternatives are "not working", are given by:

(A.7) $$P^H(j,h,0,0) = 1\{y_j(h) > y(0)\} \int_{y(0)}^{y_j(h)} \frac{g_j(h^0)\theta_j v(f^0(hw_k^0, I), h) v(f(0, dy), 0)}{K(y)^2},$$

(A.8) $$P^H(0,0,k,\tilde{h}) = 1\{y(0) > y_k(\tilde{h})\} \int_{y_k(\tilde{h})}^{y(0)} \frac{g_k(\tilde{h})\theta_k v(f^0(0,I),0) v(f(hw_k, dy))}{K(y)^2},$$

(A.9) $$P^H(j,h,j,h) = \frac{g_j(h)\theta_j v(f^0(hw_k^0, I), h)}{K(y_j(h))},$$

for positive h and j, and

(A.10) $$P^H(0,0,0,0) = \frac{v(f^0(0,I),0))}{K(y(0))}$$

for $j = h = 0$.

To calculate the compensated elasticity, we have to sum the transitions probabilities from one initial state to all possible states. Let $\varphi_k^H(h)$ be the probability that the agent chooses (k,h) ex post, given that utility is the same as before the reform. Then for $h > 0$,

(A.11) $$\varphi_k^H(h) = \sum_{j \neq k} \sum_{x \in D \setminus \{h\}} P^H(j,x,k,h) + \sum_{x \in D \setminus \{h\}} P^H(k,x,k,h) + \sum_{j \neq k} P^H(j,h,k,h) + P^H(k,h,k,h),$$

for h, $k > 0$, and

(A.12) $$\varphi^H(0) = \sum_{x \in D} \sum_{j > 0} P^H(j,x,0,0) + P^H(0,0,0,0).$$

The compensated change, for all (h, k) is given by

(A.13) $$\varphi_k^H(h) - \varphi_k^0(h).$$

And the relative change is:

(A.14) $$\frac{\varphi_k^H(h) - \varphi_k^0(h)}{\varphi_k^0(h)}$$

Appendix B. Data, tax functions and estimates

Data on the labor supply of married women in Norway used in this study consist of a merged sample of the "Survey of Income and Wealth, 1994" and the "Level of living conditions, 1995" (Statistics Norway, 1994 and 1995, respectively). Data cover married couples as well as cohabiting couples with common children. The ages of the spouses range from 25 to 64. None of the spouses is self-employed and none of them is on disability or other type of benefits. A person is classified as a wageworker if their income from wage work is higher than their income from self-employment. All taxes paid are observed and in the assessment of disposable income, at hours not observed, all details of the tax system are accounted for. Hours of work are calculated as the sum of hours of the main job as well as those of any side jobs. A large majority of the women have only one job.

Wage rates above NOK 350 or below NOK 40[6] are not utilized when estimating the wage equations. The wage rates are computed as the ratio of annual wage income to hours worked. When computing annual wage income, we take into account the fact that some women have multiple jobs. The size of the sample used in estimating the labor supply model is 810. Descriptions of variables and summary statistics are given in Table B.1.

Table B.1. Descriptive statistics, number of observations = 810 (values in NOK, 1994)

Variable	Mean	Std. Dev.	Min	Max
Number of children (age 0-2)	0.23	0.45	0.00	2.00
Number of children (age 0-6)	0.54	0.77	0.00	3.00
Number of children (age 3-6)	0.30	0.56	0.00	3.00
Number of children (age 7-17)	0.66	0.85	0.00	4.00
Age in year (men)	42.80	9.17	25.00	66.00
Education in year (men)	12.05	2.49	9.00	19.00
Age in year (women)	40.07	9.04	25.00	64.00
Education in year (women)	11.61	2.15	9.00	17.00
Sector (1=Public, 2=Private)	1.34	0.61	0.00	2.00
Work experience (woman age – woman education in years)	22.45	9.63	2.00	49.00
Capital income (child allowances included)	32306.71	42378.48	0.00	568403.00
Child allowances	13094.37	12154.01	0.00	60084.00
Women wage income per year	149751.97	83060.53	0.00	581693.00

[6] As of June 2013, 1 USD≈NOK 5,80

Men wage income per year	274372.89	106239.67	17312.00	1184861.00
Woman hourly wage in public sector	89.36	12.09	64.88	132.34
Woman hourly wage in private sector	109.77	13.68	80.14	156.44

Table B.2. Tax function in 1994 for a married nonworking woman whose husband is working, 1994.

Male income, Y_{male}	Tax T
0–41907	0
41907–140500	$0.302Y_{male}–12656$
140500–252000	$0.358Y_{male}–20524$
252000–263000	$0.453Y_{male}–44464$
263000–	$0.495Y_{male}–55510$

Table B. 3. Tax function in 1994 for a married working woman or man, NOK 1994

Wage income, Y	Tax T
0–20954	0
20954–140500	0.302Y–6328
140500–208000	0.358Y–14196
208000–236500	0.453Y–33956
236500–	0.495Y–43889

Table B.4. Estimates

Variables	Parameters	Estimates	t-values
Preferences:			
Consumption:			
Exponent	α_1	0.64	7.6
Scale 10^{-4}	α_2	1.77	4.2
Subsistence level C_0 in NOK per year		60 000	
Leisure:			
Exponent	α_3	-0.53	-2.1
Constant	α_4	111.66	3.2
Log age	α_5	-63.61	-3.2
(log age)^2	α_6	9.2	3.3
# children 0-6	α_7	1.27	4.0
# children 7-17	α_8	0.97	4.1
Consumption and Leisure, interaction	α_9	-0.12	-2.7
Subsistence level of leisure in hours per year		5120	
The parameters θ_1 and θ_2; $\log\theta_j = f_{j1} + f_{j2}S$			
Constant, public sector (sector 1)	f_{11}	-4.2	-4.7
Constant, private sector (sector 2)	f_{21}	1.14	1.0
Education, public sector (sector 1)	f_{12}	0.22	2.9
Education, private sector (sector 2)	f_{22}	-0.34	-3.3
Opportunity density of offered hours, $g_{k2}(h)$, k=1,2			
Full-time peak, public sector (sector 1)*	$\log(g_{12}(h_{Full}) / g_{12}(h_0))$	1.58	11.8
Full-time peak, private sector (sector 2)	$\log(g_{22}(h_{Full}) / g_{22}(h_0))$	1.06	7.4
Part-time peak, public Sector	$\log(g_{12}(h_{Par}) / g_{12}(h_0))$	0.68	4.4
Part-time peak, private Sector	$\log(g_{22}(h_{Par}) / g_{22}(h_0))$	0.8	5.2
# observations		824	
Log likelihood		-1760.9	

* The notation h_0 refers to an arbitrary level of hours of work different from full-time and part-time hours.

Appendix C. Elasticities

M=uncompensated wage elasticity (Marshall), S=compensated wage elasticity (Slutsky), I= income elasticity

Table C.1. Married woman aged 30, no children

Wage rate NOK 70

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	0.007	-0.043	0.163	0.152	0.222	0.163	0.160	0.178
	S	0.131	0.135	0.111	0.186	0.174	0.254	0.318	0.310	0.365
	I	0.000	-0.007	0.040	-0.060	-0.057	-0.081	-0.060	-0.064	-0.041
100000	M	0.029	0.057	-0.089	0.269	0.254	0.352	0.299	0.306	0.259
	S	0.183	0.205	0.061	0.254	0.239	0.335	0.438	0.445	0.397
	I	-0.022	-0.031	0.028	-0.080	-0.076	-0.101	-0.102	-0.107	-0.073
200000	M	0.119	0.149	-0.038	0.355	0.338	0.441	0.479	0.493	0.401
	S	0.276	0.324	0.015	0.319	0.305	0.389	0.596	0.629	0.404
	I	-0.046	-0.054	-0.009	-0.062	-0.060	-0.074	-0.109	-0.114	-0.083
400000	M	0.198	0.227	0.053	0.363	0.348	0.440	0.569	0.583	0.496
	S	0.330	0.393	-0.009	0.329	0.318	0.373	0.659	0.712	0.363
	I	-0.030	-0.031	-0.024	0.011	0.010	0.016	-0.018	-0.020	-0.007

Wage rate NOK 200

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.046	0.245	0.192	0.190	0.194	0.192	0.142	0.445
	S	0.295	0.237	0.588	0.224	0.223	0.222	0.519	0.461	0.810
	I	0.000	0.001	-0.007	-0.008	-0.008	-0.009	-0.008	-0.007	-0.017
100000	M	0.000	-0.043	0.231	0.207	0.203	0.220	0.207	0.158	0.457
	S	0.272	0.219	0.546	0.221	0.219	0.225	0.493	0.438	0.771
	I	0.000	0.001	-0.007	-0.012	-0.011	-0.014	-0.012	-0.010	-0.021
200000	M	0.000	-0.039	0.202	0.216	0.211	0.239	0.216	0.172	0.453
	S	0.240	0.194	0.481	0.215	0.212	0.227	0.456	0.406	0.709
	I	0.000	0.000	0.000	-0.010	-0.010	-0.013	-0.010	-0.010	-0.013
400000	M	0.000	-0.033	0.181	0.222	0.216	0.250	0.223	0.182	0.437
	S	0.201	0.162	0.401	0.205	0.201	0.222	0.407	0.364	0.624
	I	0.000	-0.003	0.018	0.005	0.005	0.004	0.005	0.001	0.023

Wage rate NOK 300

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.049	0.229	0.178	0.184	0.147	0.178	0.134	0.380
	S	0.347	0.297	0.574	0.169	0.175	0.138	0.516	0.472	0.713
	I	0.000	0.001	-0.006	-0.004	-0.005	-0.004	-0.004	-0.003	-0.010
100000	M	0.000	-0.051	0.240	0.186	0.192	0.155	0.186	0.140	0.399
	S	0.338	0.286	0.570	0.173	0.179	0.142	0.511	0.465	0.713
	I	0.000	0.002	-0.009	-0.007	-0.007	-0.006	-0.007	-0.005	-0.015
200000	M	0.000	-0.052	0.249	0.195	0.200	0.165	0.195	0.147	0.418
	S	0.315	0.263	0.549	0.175	0.181	0.146	0.490	0.444	0.695
	I	0.000	0.001	-0.008	-0.007	-0.007	-0.006	-0.007	-0.005	-0.015
400000	M	0.000	-0.051	0.243	0.196	0.201	0.168	0.196	0.148	0.416
	S	0.269	0.221	0.491	0.168	0.173	0.142	0.438	0.395	0.634
	I	0.000	0.000	0.003	0.002	0.002	0.001	0.002	0.001	0.005

Table C.2. Woman aged 30, two children

Wage rate NOK 70

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.047	0.096	-0.190	0.462	0.451	0.498	0.511	0.551	0.299
	S	0.327	0.394	-0.012	0.494	0.481	0.536	0.821	0.876	0.523
	I	-0.046	-0.059	0.014	-0.131	-0.128	-0.137	-0.177	-0.187	-0.122
100000	M	0.254	0.304	0.021	0.593	0.589	0.595	0.862	0.911	0.618
	S	0.581	0.704	-0.017	0.667	0.675	0.569	1.249	1.379	0.551
	I	-0.126	-0.137	-0.075	-0.149	-0.148	-0.146	-0.273	-0.284	-0.221
200000	M	0.511	0.554	0.319	0.634	0.637	0.603	1.177	1.227	0.942
	S	0.682	0.830	-0.027	0.410	0.382	0.508	1.093	1.212	0.480
	I	-0.167	-0.173	-0.141	-0.111	-0.112	-0.105	-0.277	-0.283	-0.246
400000	M	0.626	0.661	0.474	0.585	0.591	0.547	1.249	1.291	1.048
	S	0.083	0.109	-0.035	0.436	0.440	0.410	0.519	0.549	0.374
	I	-0.108	-0.107	-0.110	-0.015	-0.015	-0.017	-0.123	-0.122	-0.127

Wage rate NOK 200

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	0.001	-0.006	0.201	0.188	0.277	0.201	0.189	0.271
	S	0.246	0.232	0.325	0.212	0.200	0.287	0.458	0.432	0.613
	I	0.000	-0.001	0.009	-0.014	-0.013	-0.020	-0.014	-0.015	-0.011
100000	M	0.000	0.008	-0.048	0.231	0.216	0.319	0.231	0.222	0.270
	S	0.247	0.241	0.281	0.227	0.213	0.310	0.475	0.455	0.591
	I	0.000	-0.003	0.018	-0.025	-0.020	-0.034	-0.025	-0.027	-0.016
200000	M	0.001	0.019	-0.101	0.278	0.261	0.381	0.280	0.281	0.276
	S	0.249	0.253	0.221	0.254	0.238	0.347	0.503	0.492	0.568
	I	0.000	-0.006	0.027	-0.035	-0.032	-0.046	-0.035	-0.038	-0.019
400000	M	0.009	0.035	-0.132	0.327	0.307	0.437	0.337	0.344	0.299
	S	0.243	0.259	0.159	0.285	0.267	0.383	0.529	0.526	0.542
	I	-0.003	-0.008	0.023	-0.025	-0.024	-0.032	-0.029	-0.033	-0.009

Wage rate NOK 300

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.053	0.296	0.229	0.223	0.252	0.229	0.169	0.556
	S	0.440	0.367	0.826	0.273	0.270	0.284	0.714	0.637	1.110
	I	0.000	0.001	-0.005	-0.006	-0.006	-0.007	-0.006	-0.005	-0.013
100000	M	0.000	-0.050	0.282	0.234	0.228	0.264	0.234	0.176	0.553
	S	0.413	0.345	0.778	0.268	0.263	0.284	0.681	0.608	1.063
	I	0.000	0.001	-0.006	-0.010	-0.009	-0.012	-0.010	-0.008	-0.019
200000	M	0.000	-0.045	0.253	0.244	0.235	0.284	0.244	0.189	0.544
	S	0.375	0.315	0.702	0.260	0.253	0.285	0.635	0.568	0.988
	I	0.000	0.000	-0.002	-0.013	-0.012	-0.016	-0.013	-0.011	-0.019
400000	M	0.000	-0.037	0.212	0.259	0.249	0.310	0.259	0.210	0.530
	S	0.331	0.280	0.605	0.252	0.244	0.286	0.583	0.525	0.892
	I	0.000	-0.002	0.010	-0.007	-0.007	-0.010	-0.007	-0.009	0.000

Table C.3 Married woman, aged 40, no children

Wage rate NOK 70

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	0.019	-0.078	0.187	0.172	0.249	0.187	0.192	0.168
	S	0.148	0.164	0.083	0.211	0.195	0.283	0.360	0.359	0.366
	I	0.000	-0.012	0.050	-0.076	-0.070	-0.098	-0.076	-0.082	-0.048
100000	M	0.045	0.085	-0.107	0.319	0.298	0.399	0.366	0.384	0.287
	S	0.225	0.271	0.042	0.303	0.282	0.380	0.582	0.553	0.422
	I	-0.034	-0.049	0.022	-0.100	-0.094	-0.120	-0.134	-0.143	-0.098
200000	M	0.177	0.226	-0.004	0.417	0.395	0.490	0.601	0.631	0.486
	S	0.364	0.458	0.000	0.389	0.372	0.433	0.753	0.831	0.433
	I	-0.069	-0.080	-0.030	-0.078	-0.075	-0.088	-0.147	-0.155	-0.119
400000	M	0.289	0.329	0.120	0.422	0.403	0.483	0.718	0.746	0.615
	S	0.446	0.570	-0.023	0.407	0.399	0.405	0.854	0.970	0.383
	I	-0.046	-0.046	-0.042	0.005	0.004	0.008	-0.040	-0.042	-0.034

Wage rate NOK 200

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.053	0.209	0.199	0.192	0-218	0.199	0.138	0.432
	S	0.291	0.220	0.563	0.229	0.224	0.240	0.521	0.445	0.804
	I	0.000	0.001	-0.005	-0.009	-0.009	-0.011	-0.009	-0.007	-0.016
100000	M	0.000	-0.049	0.193	0.209	0.200	0.236	0.209	0.150	0.434
	S	0.270	0.205	0.517	0.226	0.220	0.244	0.498	0.426	0.762
	I	0.000	0.001	-0.004	-0.013	-0.013	-0.016	-0.013	-0.011	-0.021
200000	M	0.000	-0.041	0.165	0.223	0.212	0.260	0.223	0.170	0.430
	S	0.241	0.187	0.450	0.223	0.214	0.249	0.464	0.402	0.699
	I	0.000	0.000	0.003	-0.014	-0.013	-0.018	-0.014	-0.013	-0.014
400000	M	0.001	-0.033	0.135	0.236	0.223	0.280	0.237	0.189	0.419
	S	0.207	0.163	0.372	0.217	0.208	0.249	0.424	0.372	0.622
	I	0.000	-0.005	0.018	0.001	0.001	0.000	0.000	-0.003	0.018

Wage rate NOK 300

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.070	0.258	0.200	0.207	0.172	0.200	0.138	0.427
	S	0.393	0.315	0.650	0.198	0.206	0.167	0.591	0.521	0.817
	I	0.000	0.002	-0.007	-0.005	-0.005	-0.004	-0.005	-0.003	-0.011
100000	M	0.000	-0.074	0.259	0.208	0.214	0.181	0.208	0.138	0.446
	S	0.379	0.301	0.640	0.201	0.208	0.171	0.581	0.510	0.812
	I	0.000	0.002	-0.010	-0.008	-0.008	-0.007	-0.008	-0.005	-0.017
200000	M	0.000	-0.075	0.265	0.217	0.222	0.193	0.217	0.145	0.463
	S	0.351	0.276	0.610	0.202	0.209	0.175	0.554	0.483	0.786
	I	0.000	0.002	-0.009	-0.008	-0.008	-0.007	-0.008	-0.005	-0.016
400000	M	0.000	-0.073	0.256	0.219	0.223	0.198	0.219	0.149	0.460
	S	0.300	0.229	0.543	0.194	0.200	0.171	0.495	0.429	0.715
	I	0.000	-0.001	0.003	0.001	0.001	0.000	0.001	0.000	0.004

Table C.4. Married woman aged 40, two children

Wage rate NOK 70

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.065	0.131	-0.153	0.508	0.500	0.521	0.578	0.638	0.360
	S	0.371	0.474	0.011	0.566	0.557	0.571	0.938	1.031	0.582
	I	-0.065	-0.082	-0.010	-0.145	-0.143	-0.146	-0.210	-0.224	-0.156
100000	M	0.331	0.393	0.130	0.629	0.631	0.605	0.981	1.049	0.744
	S	0.714	0.920	0.006	0.784	0.820	0.576	1.498	1.740	0.582
	I	-0.163	-0.176	-0.122	-0.158	-0.160	-0.151	-0.320	-0.334	-0.271
200000	M	0.620	0.670	0.465	0.649	0.660	0.599	1.309	1.374	1.093
	S	0.821	1.063	-0.008	0.412	0.376	0.487	1.233	1.440	0.478
	I	-0.203	-0.209	-0.183	-0.116	-0.118	-0.108	-0.317	-0.325	-0.290
400000	M	0.724	0.763	0.606	0.589	0.602	0.537	1.356	1.411	1.117
	S	0.066	0.094	-0.019	0.414	0.422	0.378	0.480	0.517	0.358
	I	-0.130	-0.129	-0.132	-0.021	-0.021	-0.023	-0.151	-0.150	-0.155

Wage rate NOK 200

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	0.014	-0.062	0.218	0.199	0.297	0.218	0.214	0.233
	S	0.257	0.253	0.273	0.223	0.204	0.301	0.480	0.458	0.575
	I	0.000	-0.003	0.013	-0.018	-0.016	-0.024	-0.018	-0.019	-0.011
100000	M	0.000	0.026	-0.105	0.257	0.235	0.348	0.258	0.265	0.239
	S	0.265	0.272	0.234	0.245	0.225	0.332	0.511	0.497	0.567
	I	0.000	-0.005	0.023	-0.031	-0.028	-0.041	-0.031	-0.034	-0.018
200000	M	0.002	0.042	-0.156	0.316	0.291	0.419	0.319	0.335	0.255
	S	0.273	0.295	0.181	0.284	0.261	0.380	0.557	0.557	0.561
	I	-0.001	-0.009	0.032	-0.044	-0.040	-0.056	-0.045	-0.050	-0.024
400000	M	0.0156	0.065	-0.178	0.375	0.348	0.480	0.391	0.415	0.294
	S	0.273	0.311	0.123	0.326	0.301	0.424	0.600	0.613	0.547
	I	-0.006	-0.013	0.022	-0.035	-0.033	-0.042	-0.041	-0.046	-0.019

Wage rate NOK 300

Income I	Elasticity	El: probability of working			El: conditional hours			El: unconditional hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.059	0.244	0.221	0.211	0.255	0.221	0.149	0.506
	S	0.419	0.331	0.765	0.267	0.259	0.292	0.687	0.590	1.057
	I	0.000	0.000	-0.003	-0.006	-0.006	-0.008	-0.006	-0.005	-0.012
100000	M	0.000	-0.054	0.225	0.228	0.216	0.269	0.228	0.160	0.501
	S	0.395	0.315	0.715	0.261	0.252	0.292	0.657	0.567	1.008
	I	0.000	0.000	-0.003	-0.011	-0.010	-0.014	-0.011	-0.009	-0.018
200000	M	0.000	-0.046	0.190	0.243	0.228	0.295	0.243	0.180	0.491
	S	0.363	0.294	0.640	0.255	0.244	0.295	0.619	0.538	0.935
	I	0.000	0.000	0.001	-0.015	-0.014	-0.020	-0.015	-0.014	-0.018
400000	M	0.000	-0.034	0.143	0.265	0.248	0.332	0.265	0.213	0.486
	S	0.329	0.274	0.549	0.253	0.240	0.302	0.582	0.514	0.852
	I	0.000	-0.003	0.012	-0.012	-0.010	-0.016	-0.012	-0.014	-0.003

Appendix D. Indifference curve and Indifference band