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OPEN INNOVATION IN A MODEL À LA HOTELLING

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Open Innovation in a Model à la Hotelling

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This paper shows a model *à la* Hotelling in which profit-maximizing firms use either an *open* or a *closed* strategy to develop their software products. Only in the first case they can freely interchange information about their R&D, and the spillovers are higher the closer they are.
What comes out is a *clustering force* that drives *open* firms to stay closer one another in the product characteristic space and which lead to believe that a sense of *community* is essential to work for an organizational model that is decentralized, modular and that cannot be planned in advance, like the *Bazaar development model*, used by *open* firms, is.

Keywords: *Hotelling's model*; *spillovers*; *copyleft*.

1 Introduction

To understand the meaning of this paper we have to keep in mind that there are profit-maximizing firms that use a knowledge management style (and software is knowledge)[1] so-called *open*,[2] achieving remarkable success.[3]

When I speak about the «*closed* model» I refer to those firms that use their Intellectual Property Rights (IPRs), obtained as a result of R&D investments, in order to get a temporary and "local" monopoly that ensures them a rent.

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[1]See, for example, Williams and Stallman, 2010, p. 19.

[2]We can think to Red Hat, IBM, SUSE (ex Novell), Sun Microsystems (now Oracle America), to name a few.

[3]For example, *Red Hat Inc.* [`http://www.redhat.com`] an American multinational software company founded in 1993 and went public in 1999, with 5700 employees in 2013, $1.13 billion in annual revenue and $147 million of net incomes during 2012 [sources: `http://investors.redhat.com/faq.cfm` and `http://investors.redhat.com/annuals.cfm`], is one of the biggest companies in the free and open-source software world.

On the other hand, I use «*open* model» referring to those firms that, to develop software products, make of goods or services complement to this one – like a smartphone or a technical support service – their main profit centre, letting the other firms in the market exploit the new knowledge produced by their investments in R&D freely. In return, they ask to get (at least in potency) a performance of opposite sign in the future,[4] not based on an equivalence relation, but on a reciprocity relation[5] that, as the way the gift works [Mauss, 1923–1924], allows to create social (stable) relationships and so to build *communities*.[6] The complexity that characterizes such an organizational structure is solved using what Eric Raymond [1998] called *Bazaar model*, an organisational model founded on decentralization, modularity and redundancy of its parts that cannot be planned in advance, using the *community* as the communication medium between the components of the system (analogously to the *territory* in the *industrial districts / business clusters* framework).[7]

The *community* element leads us to believe that the knowledge spillovers among the firms that use a Bazaar organizational model generate a *product characteristic cluster* between them, inside which *tacit local knowledge* can be found (that is, «specific public goods» [Bellandi, 2003, pp. 158–160]).[8]

Starting from this, I have analysed the Free-Software Movement as a prototype of an organizational and institutional model, identifying the *Bazaar model* and the *community* as the fundamental elements of this new *ideal type* (or *pure type*).

Therefore, I have used a model *à la* Hotelling to check if characteristic clusters such as those now described can also be found in a microeconomic (theoretical) model.[9]

The model I have built, based on the one by Mai and Peng [1999], is a two-stage game, in which three firms can use, as strategic variables, their mill prices and their locations

[4] This was obtained by the use of a *copyleft* licence [see, e.g., Free Software Foundation, 2013].

[5] A *reciprocity relation* is characterized by three elements [Bruni and Zamagni, 2004, p. 167]: a *bi-directionality* of the exchange, differently from the *pure altruism*; a «conditional-unconditionality» [Caillé, 1998] that differentiates it from an *equivalence exchange relation*, because an exchange in one direction does not constitute a prerequisite for the implementation of a further exchange of opposite sign; a *transitivity* of the exchange that so is "open", meaning that the reciprocating action of the second subject of the relation can be directed to a third party.

[6] Particularly, *imagined communities* [B. Anderson, 1991] [see also Aime and Cossetta, 2010].

[7] Contrary to what happens in the various types of "geographical districts", it is completely absent of the spatial dimension: so, speaking of «district» is only a way to call to the mind the analogy between this organizational form of production and the other *localized production process* types. For this reason, the *relational* dimension is, here, extremely relevant, while the variables that refer to the *institutional-regulatory environment* becomes meaningless, since the *non-territorialized* character of this type of production system [see Rullani, 2002 for an interesting interpretation of Industrial Clusters as Complex Adaptive Systems].

[8] «Specific public goods» are not *club goods*, because they are *non-rivalrous* and *non-excludable*, like all *public goods*. But, differently from «universal public goods», they are not necessarily free for everyone. As explained by Bellanca [2007, p. 227], it is not possible, for example, to exclude someone from the use of a technical language, but before you can use it you must learn it; and to do so you must spend time and other resources.

[9] Therefore, my trial can be seen from two standpoints. I have tried to reproduce some empirical observations in a theoretical model, but I also have attempted to give micro-foundations to a sociological and historical analysis that strengthens it.

in the (linear) characteristics space – that represents also their market –. The profit function of (only) two of them is characterized by a *spillover effect*. Compared with the "standard" Hotelling scheme[10] it has been necessary to made two changes. On one hand, I had to introduce a third firm in the market so as to have the minimum requirements to get a mixed *open-closed* market.[11] On the other hand, I added a knowledge spillover effect affecting the (two) *open* firms; the strength of this information flow is directly proportional to the distance between them in the product characteristic space, and it reduces the production costs.[12]

With the simultaneous introduction of those two changes we get a game in which two different forces counteract the *strategic effect* – typical of all the models *à la* Hotelling with quadratic transportation costs – and the resulting *centrifugal force*. In addition to the *centripetal force* shown by Kats and Neven [1990], caused by the presence of a further firm in the market that strengthens the *demand effect*, there is also a *clustering force*, which concerns only the (two) *open* firms – since it derives from the presence of the knowledge spillovers in the model –, that leads them to approach one to the other with the increasing of the exogenous spillovers level.

This latter force originates clusters around a certain products characteristic, that can be explained, from a historically-institutionalist point of view, thinking that, as we have said, within an "open system", the exchanges ground not on a relation of equivalence of value between the two terms of the exchange, but on a reciprocal relation. Such an exchange relation, establishing lasting relationships among people, sets up a *community*, that, in the model, could be represented as a product characteristic cluster.

Concluding, we can so say that the existence of a *community* is essential to the Bazaar development model to run. The *community* element gives, to the parts involved in the process, a low cost medium through which to communicate and interact, with no need of institutional rules, like contracts – as in a *market model* – or the hierarchy and the command – as in a *Fordist model* –, that facilitate the interaction between them. Some firms will be able to exploit tacit knowledge spillovers at low cost, using as a medium the *community* – founded on a sense of belonging to a past perceived as common and on a shared view of the future –. Those firms will gather round a certain technological characteristic. This exploitation, recombining pieces of already known knowledge, originates, as highlighted by Joel Mokyr [2004, pp. 147–148], non-linear effects that cannot be planned. Those effects are extremely useful to the growth and the development of an economic system, also without an enlargement of the «epistemic base of the techniques» in use.

2 Previous literature

The model I will show is grounded on two different strands of literature. One natural reference is the young literature on free and open-source software. The other is the wide

[10] With "standard" I refer to 1979 quadratic transportation cost model by d'Aspremont et al.
[11] In this respect, I have used the 1990 paper by Kats and Neven.
[12] As in Mai and Peng, 1999; Piga and Poyago-Theotoky, 2005.

literature about the models *à la* Hotelling, referring inter alia to the models with more than two firms and to the ones in which there is a spillover between some of the firms.

2.1 The economic literature about free software

Despite the fact that firms which get profits through free and open-source software development exist and that they have considerable success, the literature on the subject is still poor.

The first papers about free and open-source software tried to explain what gives individual developers the incentive to contribute to those type of projects, seemingly for free [you can see, for an excellent review, Lerner and Tirole, 2005; von Krogh and von Hippel, 2006; Schiff, 2002; Rossi, 2006; David and Shapiro, 2008]. Despite the fact that this question is extremely interesting, we must remember that a large part of the free and open-source software was developed by people that are payed to do that job.[13]

In some notable and innovative contributions of almost more than ten years ago, Lerner and Tirole [2001, 2002, 2005] pointed out the future courses of scientific research about free and open-source software.[14] They highlighted, among other things, how the question of competition between *open* and *closed* firms – according to the terminology that I have used here – had received, until that time, an inadequate attention: after ten years we can sadly say that the state of art substantially has not changed so much. This also because, even when some tries have been done, most of the times the model was built so that the *open* firms cannot benefit from their real competitive (organizational) advantage, that consists in the possibility of exploiting the spillovers that arise between them. As a matter of fact, most of the papers about competition between firms of the two paradigms use duopolistic models, which does not let the (only one) *open* firm, clearly, look realistic [Bitzer, 2004; Casadesus-Masanell and Ghemawat, 2006; Economides and Katsamakas, 2006; Gaudeul, 2007; Mustonen, 2003; Athey and Ellison, 2010; Arora and Bokhari, 2007]. Only in some more recent works, like the ones by Bessen [2006]; Casadesus-Masanell and Llanes [2011]; Haruvy et al. [2008]; Henkel [2004]; Johnson [2002]; Schmidtke [2006], *open profit-maximizing* firms have been introduced. This helps increase the adherence between the mathematical model and the historical straight lines previously shown and of solving the limit highlighted by Llanes and de Elejalde [2013, p. 38] when they said that “[i]ntroducing profit-seeking open-source firms is important

[13] For example, a key element of the free and open-source software ecology, like the Linux kernel, was developed, according to Corbet et al. [2012, p. 9, 2013, p. 9], for not less than the 75-80% by salaried workers. Furthermore, we can see that the five firms that who have made larger contributions to it were, in 2012, Red Hat with the 11.9%, Novell with the 6.4%, Intel with the 6.2%, IBM with the 6.1% and Oracle with the 2.1%, while in 2013, they were Red Hat with the 10.2%, Intel with the 8.8%, Texas Instruments and Linaro both with the 4.1% and SUSE (previously hold by Novell) with the 3.5%. Also if the data refered by Ghosh [2006], in which two-thirds of free and open-source software was developed by individual contributors, suggest an opposite feeling, we must read this discord in the light that, while the first group of data refers to a key and quite complex element of operating systems, the latter shows data about the Debian GNU/Linux distribution available packages, and so also to software elements of less relevance and less sophisticated technical level.

[14] For a more recent survey, you can see Maurer and Scotchmer, 2006; Fershtman and Gandal, 2011; Llanes and de Elejalde, 2013; von Engelhardt, 2010.

because doing so allows us to analyze the incentives for investing in R&D and the decision to become open source".

There is a green literature that studies mixed oligopolies, i.e. in which *open* and *closed* firms coexist [Llanes and de Elejalde, 2013; von Engelhardt, 2010]. Specifically, Llanes and de Elejalde, by using a multinomial logit model, show a two-stage non-cooperative game in which, in the first the firms choose whether to be *open* or *closed*, and in the second they choose the R&D investments levels and their mill price.

2.2 The economic literature about the models à la Hotelling

Differently from the latter two papers, the model that I propose here gets into the wide line of research about product differentiation, opened in 1929 by Harold Hotelling with a very often quoted paper entitled 'Stability in Competition'.

As concisely cleared by Gabszewicz and Thisse [1992, p. 282], "[s]pace, by its very nature, is a source of market power".[15] Being the competition between firms the stronger, the greater the degree of substitutability between the goods they sell is, being able to differentiate their own product from those of their competitors is a way to raise their profits over the level theoretically achievable under perfect competition. Thereby, as a matter of fact, each firm gets some degree of market power arising from the consumers' preferences for "proximity" [Carlton and Perloff, 1997; Beath and Katsoulacos, 1991; Shy, 1995, p. 133].

The *location models* are monopolistic competition models. Specifically, looking at the case in which the relevant space is a *characteristic* one, the consumers appear to have a heterogeneous pleasure.[16] Furthermore, we assume the consumer preferences are asymmetrical.[17] Finally, we must point out that the goods turn out to be different for two different reasons: on one hand, because the firms are located in different points of the space; on the other, because there are positive transportation costs involved in matching consumers and goods.[18]

[15] As already shown by Harold Hotelling in his 1929 original paper, it is possible to exploit the analogy between the *geographical space* and the *product characteristic space* so that we can expand the scope of this wide literature beyond the only «spatial» field, in the geographical sense usually meant.

[16] That is each of them differs about what is considered to be the ideal type within a set of available goods.

[17] To make the notion of «asymmetric preferences» clear it is helpful to think about the kind of goods as points of a multidimensional space. Then *asymmetric* means that each particular consumer will consider two adjacent types that are close to its position as very narrow substitutes, whereas the farther a type is from his/her ideal point the less it will be a substitute: i.e., in this type of models consumers are able to distinguish between different products and deal the various kinds of goods as closer but imperfect substitutes.

[18] When we are dealing with the characteristics of goods, these transportation costs can be thought of as a loss of consumers' *surplus*, as a result of a non-coincidence between the actual kind of goods and the consumer's ideal one.

Minimum and maximum differentiation Contrarily to what was supposed by Hotelling, if we take into account a quadratic transportation cost function[19] like $t(l - x_i)^2$, spatial competition does not lead to *minimum differentiation*,[20] once you have taken into account the price choice.[21] This absolutely disagrees with the pure space competition model, that leads to the result of «minimal differentiation» between the two firms [v. S. Anderson et al., 1992, pp. 276–280]. The «principle of minimum differentiation» cannot hold under price competition, since, if the firms choose the same location as the others', they will fall into an undesirable price war *à la* Bertrand. Therefore, in equilibrium, instead of a cluster, the firms want to section the market, in order to achieve some degree of market power against the consumers located in their neighbourhood. As a matter of fact, so doing the firms can have a positive demand with a positive (mill) price, and so get a positive profit, also if the opponents charge a zero price [S. Anderson et al., 1992, pp. 298-299].

Demand effect and strategic effect Therefore, in the models *à la* Hotelling two effects of opposite sign are borne. On one side, the *demand effect* induces the firms to locate close to one another. On the other, the *strategic effect* induces them to differentiation. The relative strength of the two effects, in the simplest model, depends both on the transportation cost level, t, and on the consumers distribution [you can see, for example, Cabral, 2000, pp. 215–217; Tirole, 1991, p. 488; S. Anderson et al., 1992, pp. 299–300; Garella and Lambertini, 2002, pp. 345–348].

This trade-off has a key relevance, not only in the reading of the outcomes of the model, but also because, being at the basis of the *principles of minimum and maximum differentiation*, it has led to the growth of this branch of literature. Indeed, from the paper by d'Aspremont et al. [1979], scientists have tried to re-establish the validity of the first of the two principles. Such a try was carried forward by the introduction of parameters further than those in the simplest model, that "overturned" the relative strength of the two effects.[22] Specifically, in the model that I have developed, I have used two of those "variations": I have added a third firm in the market, and I have introduced a knowledge spillover effect between the two *open* firms.

The first of the two innovations was suggested in 1990 by Kats and Neven. In this model there is only one, unexpected, *location-then-price* equilibrium obtained in which

[19] The quadratic cost assumption is an artifice, used in many papers, like those most directly connected to the model here shown [Kats and Neven, 1990; Mai and Peng, 1999], because it guarantees the existence of an equilibrium in the price stage, as proved by d'Aspremont et al. [1979]. However, we must bear in mind that this condition is neither necessary nor sufficient to guarantee such an equilibrium [S. Anderson et al., 1992, p. 161 and 293–297; Brenner, 2001, tab. 1 p. 10; Gabszewicz and Thisse, 1992, p. 288].

[20] In particular, we can say that it leads to *maximum differentiation*, but only if we impose to the firms not to locate outside the market. In the opposite and more general case, analysed by Lambertini [1994]; Tabuchi and Thisse [1995], it is not possible to identify a *maximum* differentiation level as it can reach an infinite level.

[21] Other cases in which it happens are shown by Economides [1986]. A general argumentation can be found in d'Aspremont et al. [1983]; S. Anderson et al. [1992].

[22] An illustrative list of some parameters used in the literature can be found in Mai and Peng [1999, pp. 463–464].

the two peripheral firms are located in $x_1 = \frac{L}{8}$ and $x_3 = \frac{7L}{8}$ and the inner firm in $x_2 = \frac{L}{2}$, where L is the length of the market. Here, the peripheral firms are located inside the market also without any constrain to their locations, which could be surprising because in the standard model their optimal locations are in $x_1 = -\frac{1}{4}$ and $x_3 = \frac{5}{4}$, beyond the market bounds, as shown by Lambertini [1994]; Tabuchi and Thisse [1995]: we can explain why it happens, using the trade-off previously analysed. In the two-firms model, the price competition effect is everywhere dominant. On the contrary, in the three-firms model the equilibrium price is significantly lower and decreases the slower, the more the peripheral firms approach the centre of the market. This implies a lower strength of price competition, that gets to be dominated by the market-share effect when the peripheral firms are located near the bounds of the I interval [S. Anderson et al., 1992, pp. 299–300]. So, since the equilibrium price of the inner firm is lower than the others, as it faced a competition pressure on both sides, the model proposed by the two scientists forecast that one of the firms supplies a "median" good, with a lower markup and with a wider production scale, of about half the market. The consumers with special tastes will be supplied at a higher price by "particular" firms [S. Anderson et al., 1992, pp. 298-301].

The latter change was proposed in 1999 by Mai and Peng. It consists in the introduction of a spillover effect inversely proportional to the distance between the firms like this

$$\tau(x_2 - x_1)^2.$$

The game resolution gives rise to a *centripetal force*, inversely proportional to the ratio of exogenous transportation cost level to the exogenous externalities level $(\frac{t}{\tau})$. This force was able to counteract the *centrifugal force* coming from the price competition.

So, in the model by Mai and Peng [1999] the *distance* has a further task, beyond that to let the firms to keep some market power level. It also ensures that the R&D investments of each firm be, in some part, *specific.*[23] This type of investments cannot be reused to produce goods with characteristics that are different from those for which they were conceived.

3 The model

According to the previous literature, examined in section 2, and referring in particular to the paper by Mai and Peng [1999], I will show here an *address model* with quadratic transportation costs, in which three firms produce a homogeneous good, rivalling in the (mill) prices, each using, as a further strategic variable, its own location in the product characteristic space.

[23] As shown by authors like Nelson and Winter [1982]; Dosi et al. [1988], a portion of useful knowledge is *firm-specific.* This portion is a *non-free* «public good» [Rosenberg, 1990; Pavitt, 1991], in the meaning explained by Bellandi [2003] with the expression «specific public goods». See n. 8 on page 2. This means that firms invest in knowledge production also to understand and assess better the external knowledge [Cohen and Levinthal, 1989]: as highlighted by Gambardella and Pammolli [2000, p. 155], the skill of a firm to successfully use "public" pieces of knowledge is higher when the firm carries out research on similar subjects.

3.1 Model Description

The profit function of two of these firms is characterized by a spillover that, as in Mai and Peng [1999] and in Piga and Poyago-Theotoky [2005], depends on their relative location in the space of the characteristics of the product: they represent two *open* firms.

The third one is, instead, a *closed* firm, that uses IPRs in the standard way to capture completely the profit flow coming from the innovation it has obtained by some investments in R&D. On the other side, the third firm, due to this choice, cannot take any advantage of other firms' innovations: it shows, i.e., a zero spillover rate; both because its innovations cannot be exploited by rival firms freely and because it cannot take advantage of other firms' innovations freely.[24]

The firms The three firms produce a homogeneous good at an equal marginal cost $c(F_0)$, which is a function of sunk fixed costs, F_0, with $c^{'}(F_0) < 0$: we can think of F_0 as an irreversible investment in cost-reducing R&D.

It is assumed that cooperation between *open* firms (spillovers) takes, in the model, a particular form: information about R&D is exchanged through communication with one another.[25] A closer relative spatial proximity between the two firms makes the reciprocal information about R&D exchange more simple and, in this way, it reduces R&D costs: In this respect, the R&D costs decrease with the distance between the two firms. In the following, we define $\tau(x_i - x_j)^2$ as the total communication cost due to an exchange of information about R&D, where $\tau \geq 0$ is the externalities (exogenous) parameter – or communication cost per unit. Indeed, different industries may involve different degrees of externalities: the larger the value of τ, the higher the externalities between two *open* firms in the industry (being equal the relative proximity between them). The level of τ could also be thought as the degree of "freedom" (*copyleft*) of the licence chosen by *open* firms: a GNU GPL licence, for example, rather than a BSD licence [Llanes and de Elejalde, 2013, p. 38].[26]

This means that, if the two firms are located far away one from the other in the space of the product characteristics – let's say at the ends of the market –, they find more expensive to exchange information about their own R&D efforts mutually: we can think that this happens, for example, because these efforts are, partially, specific and linked

[24]To make the analysis easier, we imagine here that it is not possible to produce an *open/closed* mixed code.

[25]It is further assumed that firms symmetrically communicate and they equally split communication cost.

[26]There is a wide range of ways in which you can manage the exclusive rights to the use and distribution of an original work. One possible classification refers to its compliance with the *copyleft* principle [Free Software Foundation, 2013]. Specifically, we can define a kind of gradation of the *copyleft* concept by reason of the manner in which it spreads to derivative works. The principle will be stronger when the licence provides that all derivative work must be released under the same licence (e.g., the GNU GPL). While, there are less stronger cases in which only some works are subjected to this criterion (e.g., the GNU LGPL or the Mozilla Public License). Lastly, there are non-*copyleft* cases in which the derivative works are not subject to these restrictions (e.g., there are *open-source* licences, that are not *copyleft*, like the BSD, the MIT or the Apache; which allows to produce *open-source/closed-source* mixed code.

to a particular characteristic of the product sold by one of the firms involved but not by the other. On the other side, if the two firms decide to sell products that are similar in their main feature, it could be easier for the two firms make the outcomes obtained by their R&D efforts known one to the other.

Without loss of generality, we assume $x_1 \leq x_2 \leq x_3$ throughout this paper. However we must separate two cases and analyse them one after the other. First we will see the one in which the *open* firms are peripheral, that is firms 1 and 3 [sec. 3.2.1]. Secondly, instead, we will see the case in which the *open* firms are located one near the other, that is firms 1 and 2 or 2 and 3, likewise [sec. 3.2.2].

The profit function of the i-th firm is given by

$$\pi_i = (p_i - c(F_0))\mathcal{D}_i - (F_0 + \alpha\tau(x_j - x_i)^2), \tag{1}$$

where α is a dummy variable that takes the value 0 when the i-th firm is a *closed* one and 1 otherwise and where

$$\mathcal{D}_i = \begin{cases} \hat{l}_{12}, & \text{for } i = 1; \\ \hat{l}_{23} - \hat{l}_{12}, & \text{for } i = 2; \\ 1 - \hat{l}_{23}, & \text{for } i = 3. \end{cases} \tag{2}$$

For example, we can think about firms that trade smartphones different one another in their CPU family (for example, *ARM* or *Intel*). Each firm develops also an operating system – complementary good to the first, on which the R&D efforts have an influence – that is sold jointly with the electronic device so that it can work. The more similar the smartphones sold by the two *open* firms are one another, the more easily the R&D efforts – about the operating system the hardware is equipped with – of one of the firms can be exploited and with similar performances by the other;[27] moreover, the higher the parameter τ, the more it will be possible to exploit the *spillovers* arising from the proximity to the other firm – for example, if we read, as I have written earlier, such a parameter as the degree of "freedom" guaranteed by the type of licence chosen by the two firms (*copyleft*). By this view, the R&D efforts focus on some goods that generate spillovers, depending on the distance between the two *open* firms and on the degree of "openness" (*copyleft*) of the licence under which such software has been released. The product differentiation applies to one of its complement goods – a hardware component or a service, for example – and the position of the other firms, here, matters only in a strategic sense, not entering the functional form directly. Moreover, and for this reason, there are no non-linear effects connected to these differences.

The consumers There is a continuum of consumers uniformly distributed on a unit-length interval, with a mass normalized to unity, without loss of generality. Each consumer decides to buy one unit of the good from the firm which makes the lower full price

[27] It might happen, for example, that the two *open* firms adopt two different types of a same CPU family. In that case, although some shares of the opponent's R&D efforts can be exploited by the other firm, the outcomes obtained will certainly have lower performances, higher energy consumption and so on.

– that is composed of the mill price p_i plus another part depending on the transportation cost, $t > 0$, and on the distance between the location of the firm and the consumer $(l - x_i)$, where x_i, $i = 1, 2, 3$, is the location of the i-th firm –, if the full price does not exceed the reservation price of the consumer himself.

A consumer located at $l \in [0, 1]$, who decides to buy one unit of good from i-th firm, will get a surplus

$$s - p_i - t(l - x_i)^2,$$

where $s \geq 0$ is the basic reservation utility obtained by any consumer who purchases from any of the firms and p_i is the mill price of the product of the i-th firm and $t > 0$ represents the extent of product differentiation as perceived by consumers [Piga and Poyago-Theotoky, 2005, p. 130].

The game Following the classical model proposed by Hotelling [1929], and the following large literature, I will present here a two-stage non-cooperative game where, in the first stage firms choose their location in the space of the product characteristics and, in the second they compete in prices. This sequence seems to be correct because we can think that, while the location in the space of the product characteristics is a long-period choice, the price choice may be changeable also in the short-period [S. Anderson et al., 1992, pp. 292-293; Cabral, 2000, p. 215]. Since we are searching for a subgame perfect equilibrium of the game now described we proceed by backwards induction, starting with the second-stage subgame.

3.2 Model resolution

Let's now show the resolution of the model, considering separately the two cases previously indicated; at the end of each section some partial considerations will be proposed.

3.2.1 The symmetric case

Let's consider firstly the case in which the *open* firms are both peripheral.

The first step is to derive the demand functions for the three firms. To do so, it is necessary (and sufficient) to identify the "marginal" consumers' location (that is their identity), indifferent between buying from firm 1 or firm 2 and from firm 2 or firm 3, respectively, solving the following two equalities:

$$s - p_1 - t(\hat{l}_{12} - x_1)^2 = s - p_2 - t(\hat{l}_{12} - x_2)^2;$$
$$s - p_2 - t(\hat{l}_{23} - x_2)^2 = s - p_3 - t(\hat{l}_{23} - x_3)^2.$$

We get

$$\hat{l}_{12} = \frac{(p_1 - p_2)}{2t(x_1 - x_2)} + \frac{x_1 + x_2}{2}, \tag{3a}$$
$$\hat{l}_{23} = \frac{(p_2 - p_3)}{2t(x_2 - x_3)} + \frac{x_2 + x_3}{2}. \tag{3b}$$

The price stage Taking the first derivatives of Eq. (1) with respect to p_1, p_2 and p_3, setting them equal to zero and then solving the resulting equations simultaneously, we obtain the equilibrium prices: [28]

$$p_1^* = c(F_0) + \frac{t(x_1 - x_2)}{6(x_1 - x_3)}[2x_3 - 2x_2 - (x_1 - x_3)(3x_1 + 2x_2 + x_3)]; \tag{4a}$$

$$p_2^* = c(F_0) + \frac{t(x_1 - x_2)(x_2 - x_3)}{3(x_1 - x_3)}(x_1 - x_3 - 2); \tag{4b}$$

$$p_3^* = c(F_0) + \frac{t(x_2 - x_3)}{6(x_1 - x_3)}[-8x_1 - 2x_2 + 6x_3 + (x_1 - x_3)(3x_3 + 2x_2 + x_1)]. \tag{4c}$$

Now we can substitute the equilibrium prices, Eqs. (4), into the expression for profits, Eq. (1), ending the price subgame analysis.

The location stage In the first stage, the firms choose thire locations in the space of the product characteristics, anticipating how this choice will reverberate and affect their subsequent price choice.

Setting to zero the first derivative of the profit function of the second firm with respect to its location we get its best-response function:[29]

$$x_2(x_1, x_3) = \frac{x_1 + x_3}{2}.$$

Imposing such value for x_2, we can see that the derivative of the profit of firm 1 with respect to its location is zero in two points:

$$x_1(x_3) = -\frac{1}{56t}\Big(144\tau + 7t + 10tx_3 \pm \sqrt{20736\tau^2 + t^2(7 + 18x_3)^2 + 288\tau t(7 + 66x_3)}\Big).$$

But only the second of these two solutions is a maximum with respect to x_1, since only in this case $\frac{\partial^2 \pi_1}{\partial x_1^2} < 0$.[30]

Now that all the equations depend on a single relevant variable, x_3, it is sufficient, set

[28] The second order conditions are satisfied:

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{1}{t(x_1 - x_2)} < 0; \quad \frac{\partial^2 \pi_2}{\partial p_2^2} = \frac{x_1 - x_3}{t(x_1 - x_2)(x_2 - x_3)} < 0; \quad \frac{\partial^2 \pi_3}{\partial p_3^2} = \frac{1}{t(x_2 - x_3)} < 0.$$

[29] It is a maximum point with respect to x_2, as it is possible to verify that, at that location, the second derivative of the profit of firm 2 with respect of its location is negative, for $x_3 > 0$.

[30] For $t > 0$, $\tau > 0$ and $x_3 > 0$.

the locations of the other two firms

$$x_1(x_3) = -\frac{1}{56t}\Big(144\tau + 7t + 10tx_3 - \sqrt{20736\tau^2 + t^2(7+18x_3)^2 + 288\tau t(7+66x_3)}\Big), \tag{5}$$

$$x_2(x_3) = \frac{x_1 + x_3}{2} = -\frac{1}{112t}\Big(144\tau + 7t - 46tx_3 - \sqrt{20736\tau^2 + t^2(7+18x_3)^2 + 288\tau t(7+66x_3)}\Big), \tag{6}$$

to find the value by which the profit function of firm 3 is a maximum, to get the complete solution of the model.

From the FOCs, assuming $\tau > 0$, $t > 0$, $x_1 < x_3$ (with $x_1 > 0$ and $x_3 < 1$), and imposing the SOCs we get

$$x_3^* = \frac{144\tau + 27t - \sqrt{20736\tau^2 + 5472\tau t + 169t^2}}{16t}.$$

So, the best locations for the three firms are[31]

$$x_1^* = \frac{-1872\tau - 191t + 5\Psi}{448t} + \frac{\sqrt{2(7817472\tau^2 + 1700064\tau t + 51545t^2 - 49680\tau\Psi - 2691t\Psi)}}{448t}; \tag{7a}$$

$$x_2^* = \frac{2160\tau + 565t - 23\Psi}{448t} + \frac{\sqrt{2(7817472\tau^2 + 1700064\tau t + 51545t^2 - 49680\tau\Psi - 2691t\Psi)}}{896t}; \tag{7b}$$

$$x_3^* = \frac{144\tau + 27t - \Psi}{16t}, \tag{7c}$$

with $\Psi \equiv \sqrt{20736\tau^2 + 5472\tau t + 169t^2}$.

Considerations Luckily the Eqs. (7) may be reduced to a much easier form. As a matter of fact, we can see that, for $t > 0$ and $\tau \geq 0$,

$$x_1^* = 1 - x_3^* \tag{8}$$

and so, set the Eq. (6),

$$x_2^* = \frac{1}{2}. \tag{9}$$

[31] Set the Eqs. (4), the SOCs to maximize the profit functions of the three firms with respect to their relative locations are verified $\forall t > 0, \tau \geq 0$.

That is, the model leads, endogenously, to an optimal solution in the location stage where, the two peripheral firms get symmetrical locations with respect to the middle of the market,[32] and the inner firm locates, literally, in the middle of the market, regardless from the behaviour of its opponents (and of any other parameter of the model).

We, therefore, can see that, for $t > 0$ and $\tau \geq 0$,

$$\frac{\partial x_1^*}{\partial \tau} > 0, \qquad \frac{\partial x_3^*}{\partial \tau} < 0, \tag{10}$$

and that [33]

$$\lim_{\tau \to 0} x_1^* = \frac{1}{8}, \qquad \lim_{\tau \to 0} x_3^* = \frac{7}{8}, \tag{11}$$

$$\lim_{\tau \to \infty} x_1^* = \frac{1}{2}, \qquad \lim_{\tau \to \infty} x_3^* = \frac{1}{2}. \tag{12}$$

It follows that, for limit values of τ, the model can be brought back, either to the Hotelling model with three firms and quadratic transportation costs but without spillovers, or to the Bertrand model, as shown by Mai and Peng [1999] in the duopolistic case. For growing values of the exogenous spillover parameter, the firms would all be at the centre of the space of the product characteristics and prices approach the marginal cost, $c(F_0)$. On the contrary, if the exogenous spillover level decreases, the firms behave as in the model without externalities [d'Aspremont et al., 1979] and prices grow, drifting away from the marginal cost (as expected).

3.2.2 The asymmetric case

Let's now consider the case in which the two *open* firms are one next to the other, either to the right or to the left of the *closed* firm.

The price stage The resolution of the second stage of the game is the same as the previously analysed case. The equilibrium prices acquire, therefore, the same values as the symmetric case [see Eqs. (4)].

The location stage We, instead, have now to analyse the first stage of the game, in which the firms, anticipating how this choice will influence the equilibrium prices, choose their optimum locations.

[32] This outcome is coherent with the previous literature [Economides, 1989; Mai and Peng, 1999; Piga and Poyago-Theotoky, 2005], that in most cases identifies or simply analyses only the symmetric case: so, such an outcome makes the comparison with the previous results easier.

[33] I am aware that observing the behaviour of the variables x_1^* and x_3^* when the parameters approach limit values has no economical and mathematical meaning, since the profits of the *open* firms would be negative, pushing them out of the market in the long-period, due to the structure of the model [Eq. (1)]. Despite that, what I want to prove is only that, also removing the simplifying assumption $x_1 \leq x_2 \leq x_3$, firms 1 and 3 have no interest to "leapfrog", for any value of t and of τ – as clear by the Eq. (8). Even if we stress the model, switching places between the two *open* firms, they swap only their respective relative rank in the spatial ordering and the inner working of the model would converge the firms again towards the middle of the market for growing values of τ.

Unfortunately, it is impossible to get an exact algebraical solution for the best response correspondence of firms 1 and 3.[34] However, we can get an algebraical solution for the optimum response function of firm 2 which, as we will see, gives ideas that, though partial and limited, can be interesting. Therefore,

$$\frac{\partial \pi_2}{\partial x_2} = 0 \Leftrightarrow x_2(x_1, x_3) = \frac{36\tau x_1(x_3 - x_1) + t(2 - x_1 + x_3)^2(x_1 + x_3)}{2(18\tau(x_3 - x_1) + t(2 - x_1 + x_3)^2)}$$

Considerations Then, we can see that, for $x_1 < x_3$, $\tau > 0$ and $t > 0$,

$$\frac{\partial x_2(x_1, x_3)}{\partial \tau} < 0.$$

Specifically,

$$lim_{\tau \to 0} x_2(x_1, x_3) = \frac{x1 + x3}{2}; \qquad lim_{\tau \to \infty} x_2(x_1, x_3) = x_1, \tag{13}$$

that is firm 2 locates closer and closer to firm 1 as the spillover level increases (and regardless the behaviour of firm 3), while, when the exogenous parameter of the market spillover is closer to zero, firm 2 tends to move "halfway" between the other two firms. Although we cannot suppose that the two peripheral firms have a symmetrical behaviour ($x_3 \equiv 1 - x_1$), as they are not the same – in contrast to the previous case –, it is still true that for the structure of the model $\lim_{\tau \to 0} x_1^* = \frac{1}{8}$ and that $\lim_{\tau \to 0} x_3^* = \frac{7}{8}$. So, we can say, not only that for decreasing values of τ firm 2 moves away from firm 1, but also that $\lim_{\tau \to 0} x_2^* = \frac{1}{2}$; as expected.

Numerical solution for arbitrary values of the parameters Then, we can try to provide some answers that, even if not of general nature, are still interesting, giving arbitrary values to some parameters of the model.

Firstly, assuming $t = 1$, we can find the optimal locations x_1^* and x_3^* for some values of $\tau \in [0, 1]$. We can see that the values shown in Table 1 and the graph in Figure 1 (dotted lines), confirm the foregoing. As τ grows, the *open* firms approach one to the other.

With the increasing of the exogenous spillover level, firm 2 moves away from the centre of the market, so that firm 3 can get a higher market share without running into a "trying" price competition war. Therefore, the approach of firm 3 to the centre of the market must be seen in this light. The values in Tables 2, 3 and 4 show indeed that if, after getting the optimal locations of the three firms, we stressed the model setting a location for firm 3 external to the optimal one ($x_3 = \frac{7}{8}$), it would charge a price always higher than it would do having the choice about its location. Nevertheless firm 3 would get lower profits because of a squeeze of the market share otherwise controlled.

[34]For the *Abel-Ruffini theorem* about the solution of polynomial equations of degree five or higher and the *Galois theory* about solving polynomial equations by radicals, it is possible to prove that, even if all the non constant polynomial equations have always a solution (at least in the field of complex numbers), as in the *fundamental theorem of algebra*, contrary to what happens for the quadratic, cubic and quartic equations, not all polynomial of degree four or higher have an exact algebraical solution – as here it is for the derivatives of the profit functions of the two firms with respect to their own location.

Table 1: Optimal location of the firms for $t = 1$ and $\tau \in [0, 1]$ in the asymmetric case.

τ	x_1^*	x_2^*	x_3^*	$\lvert x_1^* - x_2^* \rvert$
0.0	0.125	0.500	0.875	0.375
0.1	0.224	0.492	0.847	0.268
0.2	0.269	0.484	0.832	0.215
0.3	0.296	0.478	0.823	0.182
0.4	0.314	0.473	0.816	0.159
0.5	0.327	0.469	0.812	0.142
0.6	0.337	0.466	0.809	0.129
0.7	0.345	0.463	0.806	0.118
0.8	0.351	0.460	0.804	0.109
0.9	0.356	0.457	0.803	0.101
1.0	0.360	0.455	0.802	0.095

Figure 1: Optimal locations of the firms for $t \in (0, 1]$ and $\tau \in [0, 1]$ in the asymmetric case. The graph shows, for each firm, the upper and lower borders of their sheaf of curves.

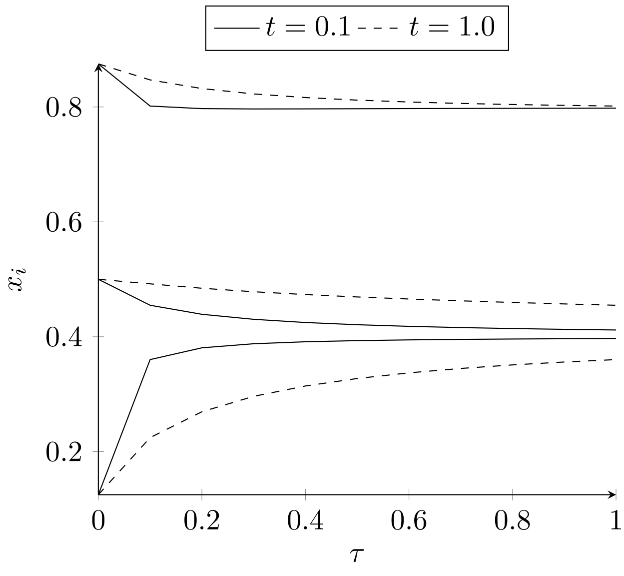

Table 2: Optimal prices of the firms for $t = 1$, $\tau \in [0, 1]$ and $F_0 = c(F_0) = 0$ in the asymmetric case.

	$x_3 = x_3^*$			$x_3 = \frac{7}{8}$		
τ	p_1^*	p_2^*	p_3^*	p_1^*	p_2^*	p_3^*
0.0	0.203	0.172	0.203	0.203	0.172	0.203
0.1	0.163	0.133	0.184	0.172	0.141	0.186
0.2	0.138	0.113	0.176	0.150	0.124	0.180
0.3	0.121	0.100	0.170	0.134	0.112	0.176
0.4	0.108	0.091	0.167	0.121	0.102	0.175
0.5	0.098	0.083	0.165	0.110	0.095	0.173
0.6	0.090	0.077	0.163	0.102	0.088	0.172
0.7	0.083	0.072	0.162	0.094	0.083	0.172
0.8	0.078	0.068	0.161	0.088	0.078	0.171
0.9	0.073	0.064	0.160	0.083	0.074	0.171
1.0	0.069	0.060	0.159	0.078	0.070	0.170

Table 3: Optimal profits of the firms for $t = 1$, $\tau \in [0, 1]$ and $F_0 = c(F_0) = 0$ in the asymmetric case.

	$x_3 = x_3^*$			$x_3 = \frac{7}{8}$		
τ	π_1^*	π_2^*	π_3^*	π_1^*	π_2^*	π_3^*
0.0	0.055	0.079	0.055	0.055	0.079	0.055
0.1	0.042	0.051	0.048	0.045	0.054	0.047
0.2	0.035	0.039	0.044	0.038	0.043	0.043
0.3	0.030	0.032	0.042	0.034	0.036	0.041
0.4	0.027	0.028	0.041	0.030	0.032	0.039
0.5	0.024	0.024	0.040	0.027	0.028	0.038
0.6	0.022	0.022	0.039	0.025	0.025	0.037
0.7	0.020	0.020	0.038	0.023	0.023	0.037
0.8	0.018	0.018	0.037	0.021	0.021	0.036
0.9	0.017	0.017	0.037	0.019	0.020	0.036
1.0	0.016	0.016	0.037	0.018	0.018	0.035

The graph in Figure 1, shows instead the upper and lower border of the sheaf of curves of the values that the optimal locations of the three firms take for values of $t \in (0, 1]$ and of $\tau \in [0, 1]$. We can see that, a decreasing of the exogenous level of the transportation costs, implies that the *open* firms tend to approach one to the other more and faster. That is, for the same exogenous spillover level, the less the exogenous level of the transportation costs is, the closer the two firms locate one to the other.

3.2.3 Leapfrogging and stability of the configuration

We can also see [Table 5] that the profits of all the firms in the asymmetric case are greater than in the symmetric one. We can so conclude that both the *open* and the *closed* firms find worthwhile to arrange themselves as in the asymmetric configuration.

In this case, the profits of the *closed* firm are always higher than those of the others. Furthermore, the comparison between the profits of the two *open* firms depends on the level of the parameter τ: for high values of τ the profits of the outer firm are higher than those of the inner firm, and conversely. Therefore, if the firms were free to choose their own location, or even to decide how to manage the flows of information arising from their innovations, as it is necessary to get a possible leapfrogging effect, then a stable configuration of the model would not exist, being understood that all the players would like to be in an asymmetric configuration.

3.3 Model analysis

To conclude, according to the short foregoing considerations for each of the two cases now analysed [sec. 3.2.1 and 3.2.2], and according to the previous literature [sec. 2.2], here I will try to outline some results that emerge from the model illustrated above.

Minimum and maximum differentiation As we have seen, the literature that followed the paper by Hotelling [1929] is largely focused, after the publication of the article by d'Aspremont et al. [1979], in trying to restore the validity of the *principle of minimum differentiation* proposed by Hotelling in his pioneering work. This attempt was carried out introducing, each time, further parameters than those used in the first one, that would produce a *centripetal* force, able to counteract the *strategic effect* that, in the duopolistic case with quadratic transportation costs and without externalities, produces a *centrifugal* force that leads to propose the *principle of maximum differentiation* hypothesis. While clearly linking up with this body of literature, the paper by Mai and Peng [1999] – and thus also this work – tries to include in the model a *centripetal* force not affecting the prices subgame, making the competition in this strategic variable less strong – as done in most parts of the former literature –, but directly the locations subgame.

As shown by Mai and Peng [1999] and by Piga and Poyago-Theotoky [2005],[35] in a duopoly *á la* Hotelling with spillovers among the firms it is possible to have equilibrium

[35]In the latter, however, a cluster of firms in the centre of the market is never a solution of the model: this because, here, both R&D is explicitly modelized and the externalities linked to the locations of the firms are endogenized [Piga and Poyago-Theotoky, 2005, p. 136].

Table 4: Differences in the market share faced by the firms if $x_3 = x_3^*$ or if $x_3 = \frac{7}{8}$, for $t = 1$, $\tau \in [0, 1]$ and $F_0 = c(F_0) = 0$ in the asymmetric case; with $\Delta_i = \mathcal{D}_i(x_3 = x_3^*) - \mathcal{D}_i(x_3 = \frac{7}{8})$.

τ	Δ_1	Δ_2	Δ_3
0.0	0.0000	0.0000	0.0000
0.1	−0.0044	−0.0047	0.0091
0.2	−0.0066	−0.0072	0.0137
0.3	−0.0078	−0.0087	0.0165
0.4	−0.0085	−0.0098	0.0183
0.5	−0.0090	−0.0105	0.0195
0.6	−0.0093	−0.0111	0.0204
0.7	−0.0095	−0.0115	0.0210
0.8	−0.0096	−0.0118	0.0214
0.9	−0.0097	−0.0120	0.0217
1.0	−0.0097	−0.0122	0.0219

Table 5: Optimal profits of the firms for $t = 1$, $\tau \in [0, 1]$ and $F_0 = c(F_0) = 0$ in the symmetic and asymmetric case.

	symmetic			asymmetric		
τ	π_1^*	π_2^*	π_3^*	π_1^*	π_2^*	π_3^*
0.0	0.0550	0.0788	0.0550	0.0550	0.0788	0.0550
0.1	0.0202	0.0295	0.0202	0.0422	0.0512	0.0477
0.2	0.0118	0.0181	0.0118	0.0349	0.0392	0.0443
0.3	0.0082	0.0131	0.0082	0.0300	0.0323	0.0422
0.4	0.0062	0.0102	0.0062	0.0265	0.0277	0.0407
0.5	0.0050	0.0084	0.0050	0.0238	0.0243	0.0396
0.6	0.0041	0.0071	0.0041	0.0217	0.0218	0.0387
0.7	0.0035	0.0062	0.0035	0.0199	0.0198	0.0380
0.8	0.0031	0.0054	0.0031	0.0184	0.0182	0.0374
0.9	0.0027	0.0049	0.0027	0.0172	0.0168	0.0369
1.0	0.0025	0.0044	0.0025	0.0161	0.0157	0.0365

solutions of firms locations that are in the middle between the «minimum differentiation» suggested by Hotelling [1929] and the «maximum differentiation» proposed by d'Aspremont et al. [1979], for intermediate values of the parameters of the model: the ratio between the communication costs and the unit transportation costs in the paper by Mai and Peng and the unit transportation costs in the paper by Piga and Poyago-Theotoky.

The outcomes I obtained in sections 3.2.1 and 3.2.2 are fully consistent with this statement. As a matter of fact, as shown in section 2.2, with three or more firms, the Hotelling model never leads to the maximum differentiation among them [Brenner, 2005; Economides, 1993; Kats and Neven, 1990]: while, i.e., for extreme values of τ, the model by Mai and Peng [1999] becomes the same as the one described by Lambertini [1994]; Tabuchi and Thisse [1995], likewise, as we are dealing here with a three or more firms oligopoly, we must expect that, for low values of τ, the model gets similar to the one shown by Brenner [2005]; Kats and Neven [1990].

The forces inside the model Therefore, in the case I have examined there are two *centripetal* forces that have to be distinguished one from the other: one resulting from the presence of the spillovers [Mai and Peng, 1999] and one resulting from the higher competitiveness coming from the presence of a third firm in the market [Brenner, 2005; Kats and Neven, 1990].[36] These two forces overlap completely and, so, can easily be confused one with the other in the symmetric case [sec. 3.2.1]. Conversely, their effects can be seen separately in the asymmetric case [sec. 3.2.2] that, therefore, although it provides only partial outcomes, is, as I said, extremely interesting in the analysis here done.

Therefore, while the first is properly a *centripetal* force, for the latter we must speak more specifically of a *clustering* force: indeed, generally this force makes those firms that take advantages of the externalities approach one to the other more than in the "standard" case; only in a particular case it has the effect of pushing them towards the middle of the market.

Then, the findings here obtained allow to perfect what highlighted by Mai and Peng. What the two authors name «centripetal force» [Mai and Peng, 1999, p. 464, 467 e 470], is actually so only by accident, also if it is surely true that the two firms have a tendency to form a cluster with the increasing of the unit communication cost, τ: in this regard I think that it may be more proper to name it «*clustering* force». The one highlighted by Kats and Neven [1990] or by Brenner [2005] with the introduction of $n \geq 3$ firms, is actually a *centripetal* force, since the firms try to locate themselves closer to the middle of the market (and not closer to the other firms) in order to increase their market share (that is, the weight of their demand). They do so regardless of the parameters of the model, t

[36] The latter is nothing more than the one that Tirole [1988] names *demand effect*. While in the case with two firms the price competition effect, that produces a *centrifugal* force on the optimal location of firms, is wherever dominant, in case of three or more firms the prices are substantially lower and go down even more if the peripheral firms approach the centre of the market. This implies that the price competition effect is less strong and is dominated by the market share effect when the peripheral firms locate near the external borders of the market [S. Anderson et al., 1992, pp. 299-300].

and τ,[37] which shows how the two effects must be considered separately, depending one on the number of firms in the market and the other on the market externalities level.

Instead, it is hard to say, if the *principle of minimum differentiation* is re-established for high values of τ, because the partiality of the outcomes got in the asymmetric case does not allow us to understand which is the optimal location of firm 3 as for the cluster of the other two firms. Logically and according of the numerical solutions obtained setting arbitrary values to the parameters of the model, it is still reasonable to expect that, for the *closed* firm, the *strategic effect* be always dominant – equals the transportation costs and the consumers distribution and given the linear-quadratic form of the transportation cost function – and that, therefore, regardless of the τ value, it is inclined to locate at some distance from the other two firms, in order not to be involved in a sharp price competition (*à la* Bertrand).

Concluding remarks The outcomes lead to say that

Proposition 1. *In an oligopoly with spillovers between only some of the firms in the market (so-called* open*):*

i *The more competitive is the market, the less (relatively) the goods sold by each firm in the market are differentiated*[38].

ii *The greater the externalities between the (two) "open" firms, the less the goods marketed by these firms are differentiated; that is, the greater the spillovers level between the "open" firms, the more they choose to sell goods mutually compatible.*

Therefore, if there are some knowledge externalities between the firms, we can see a clustering process which can be justified and, at the same time, give foundations to the idea that a "sense of community" is essential to a Bazaar (organizational) model to work.

The centrality of the *community* was not a discovery, since already Stallman [2004] said

> My hope was that a free operating system would open a path to escape forever from the system of subjugation which is proprietary software. I had experienced the ugliness of the way of life that nonfree software imposes on its users, and I was determined to escape and give others a way to escape.
>
> Non-free software carries with it an antisocial system that prohibits cooperation and community. You are typically unable to see the source code; you cannot tell what nasty tricks, or what foolish bugs, it might contain. If you don't like it, you are helpless to change it. Worst of all, you are forbidden to share it with anyone else. To prohibit sharing software is to cut the bonds of society.

[37] So much that, in the symmetric case, the reaction correspondence of the *closed* firm does not depend on the parameters of the model, but only on the locations of the competitors [Eq. (6)]: the *closed* firm locates "halfway" between the other two firms to maximize its market share. Introducing a further *closed* firm in the model ($n = 4$) it is likely that also the best response correspondence of the latter firm to be independent from the parameters of the model.

[38] see Brenner, 2005, p. 859.

But my model seems to suggest that its relevancy was not only a “moral” will (ex-ante) by developers, but also a “practical” result (ex-post) of the interaction of profit-seeking *open* firms.

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