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GROWTH MAXIMIZING GOVERNMENT SIZE AND SOCIAL CAPITAL

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Growth Maximizing Government Size and Social Capital

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Abstract

Our paper intersects two topics in growth theory: the growth maximizing government size and the role of social capital in development. We modify a simple OLG framework by introducing two key features: endogenous growth and a role for public officials in monitoring the public expenditures for intermediate goods and services supplied to private firms. Public officials have the opportunity to steal a fraction of public resources under their own control, subject to a probability of being caught and pay a fine. Hence, not all tax revenues raised by the Government reach private firms, as a fraction of them is being diverted by public officials, thus hampering growth. Under certain conditions on parameters, our main result establishes that, if the probability of detection or the fine charged on public officials who are caught stealing, or both, increase, then an increase of the tax rate is required in order to maintain an optimal growth rate, provided that also the number of public officials is increased as well. As both the probability of detection and the fine positively depend on the Social Capital level, we conclude that maximum growth rates are compatible with Big Government size, measured both in terms of expenditures and public officials, only when associated with high levels of Social Capital.

Keywords: Social Capital, Endogenous Growth, Government size, Stochastic OLG model
JEL classification: C61, O41, N9, R5

1 Introduction

The present paper intersects theoretically two topics in growth theory: the growth maximizing government size and the role of Social Capital in development. Decreasing marginal benefits of government expenditures and increasing distortions due to taxation typically leads to an inverted U relationship between growth and government size (Facchini and Melki 2011), known in the literature as B.A.R.S. curve (Barro 1989, 1990; Armey and Armey 1995; Rahn and Fox 1996; Scully 1998, 2003). Although the theoretical approach appears sound and generally accepted the optimal point from a quantitative perspective is very debated. Several empirical works did not clear cut the point over the optimal government size, typically measured as government expenditure relative to GDP. Often in this context the European Nordic Countries (ENC)[1] with large Governments and significant growth rates are taken as outliers or as counter examples to dismiss the entire approach.

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[1] Sweden, Norway and Denmark.

We believe that another stream of literature that goes under the vast and debated title of Social Capital could contain important contributions to this debate.[2] Among the many definitions of Social Capital one could find, we adhere to the one of Guiso et al. (2011), namely "those persistent and shared beliefs and values that help a group overcome the free rider problem in the pursuit of socially valuable activities". This definition of Social Capital as civicness is particularly attractive and in line with the empirical literature where several proxy of civicness appear to be related with development and government efficiency. The literature shows that greater participation to civic life and high levels of moral stigma for uncivic behavior are important factors to explain corruption and Government efficiency (Bjørnskov 2003). In particular we believe that those unwritten social norms might affect the behavior of public officials. However, we depart from the standard modeling approach which include some utility costs in the utility function due to social stigma, as for example in Guiso et al. (2004). Our idea, instead, is that a society with high levels of civicness is one where corruption and rent seeking behaviors are not tolerate easily. High social capital as understood as high trust in people and institutions as well as great level of participation in civic life are thought to foster reporting of public officials' wrongdoings to public authorities by whistle blowers. Therefore, a positive relationship between the probability of being detected—or the fine to be paid in case of detection—of dishonest public officials and the degree of civicness is expected. We consider this approach as complementary to the utility one and somehow less exposed to criticism implied by an ad-hoc form of utility function.

In more detail, we modify a simple OLG framework (Chapt. 3 in Barro and Sala-i-Martin 2004) by introducing two key features: endogenous growth à la Barro (1990) type and a role for public officials in monitoring the public expenditures for intermediate goods and services supplied to private firms. Specifically, there are two types of workers, private workers employed in competitive production sector behaving in a standard fashion, and public officials who have the opportunity to steal a fraction of public resources under their own control, subject to a positive probability of being caught and pay a fine. As a consequence, not all the stock of tax revenues raised by the Government reaches private firms as intermediate goods and services, since a fraction of it is being diverted by public officials, thus hampering growth.

As expected, we find that the endogenous growth rate of the economy is affected by the probability of detection and the fine paid by those public officials who are being caught. Moreover, as in Barro (1990), along the BGP the output growth rate turns out to be an Inverted U-Shaped function of the tax rate, thus establishing uniqueness of the optimal tax rate with respect to growth. By performing comparative dynamics on the optimal tax rate, our main result shows that, under certain conditions, if the probability of detection or the fine charged on public officials who are caught stealing, or both, increase, then an increase of the optimal tax rate is required in order to keep the growth rate at its maximum level, provided that also the share of public workers on the total workforce is adjusted (increased) as well. As both the probability of detection and the fine positively depend on the Social Capital level, we conclude that maximum growth rates are compatible with Big Government size (both in terms of expenditures and public officials) only when associated with high levels of Social Capital. When Social capital is low the growth maximizing government size shrinks and vice versa. Social Capital therefore could be the missing dimension accounting for the controversial empirical results on this issue as well as for the ENC case. According to the present model, the highest growth rates experienced by the ENC, despite their well above-the-average OECD countries' Government size, it could be explained by their highest level of Social Capital, which in turn affects the behavior of public officials and thus the efficiency of Government as a whole

This short paper proceeds as follows. In Section 2 we formally introduce the OLG framework by describing in detail the competitive firms' optimal strategies, the static general equilibrium with

[2] A survey on this large literature is beyond the scope of this paper (for a survey see, *e.g.*, Alesina and Giuliano 2013).

Government transfers to the firms, and the optimal behavior of both private and public workers. The latter results allow for a definition of general equilibrium that takes into account the (optimal) "stealing" choices made by the public employees. In Section 3 we define the optimal dynamics of physical capital that take into account the cheating behavior of public officials, characterize the BGP, and establish the main result that determines the positive monotonicity relationship between Social Capital and Government size, expressed both in terms of taxation level and share of public workers, necessary to keep growth at its maximum rate. In Section 4 we discuss a numerical example that illustrates our main result. Section 5 as usual concludes, while all mathematical proofs are gathered in the Appendix.

2 The Model

We consider a OLG model. Each individual belonging to the t-th cohort lives for two periods: in the first period, when she is young, she works either in the private or in the public sector and she consumes and saves part of her wage, net of taxes. In the second period, when she is old, she does not work but she consumes what she saved in the first period plus interests net of taxes. We assume that in the economy the population is constant over time. In each period we have L young workers, with L a large number, of which L_1 are employed in the public sector and L_2 in the private sector, with $L = L_1 + L_2$. Moreover, in each period the economy is populated by L old individuals belonging to the previous $t - 1$ cohort, so that, at each time t, the total population is $2L$.

Each individual has the same logarithmic instantaneous utility function, $u(c) = \ln c$, and the same (constant) pure rate of time preference, $0 < \beta < 1$. All young individuals inelastically supply one unit of labor either to private firms or to the Government. The share of workers employed in the public sector is constant and equal to $\lambda = L_1/L$, whereas the share employed in the private sector is again constant and equal to $1 - \lambda = L_2/L$, with $0 < \lambda < 1$.

2.1 Firms

Following Barro (1990) we assume that the Government supplies intermediate goods and productive services G to private firms financed through a distortionary tax, with rate $0 < \tau < 1$, on the total national income. G is assumed to be non excludable but rival and subject to congestion caused by its use by both private and public workers; hence only the share $g = G/L$ of G turns out to be available to each single firm. The representative private firm behaves competitively and produces a composite consumption good according to a Cobb-Douglas technology, so that firm-i output is given by

$$Y_i = \theta K_i^{\alpha} (gL_i)^{1-\alpha},$$

where θ is some positive constant indicating the (exogenous) technological level, K_i is physical capital, L_i is the number of workers employed, g is the share of intermediate goods and productive services provided by the Government available to firm-i, and $0 < \alpha < 1$ is the physical capital factor share.

Assuming, for simplicity, that capital does not depreciate, for given K_i, L_i and g firms maximize profit when

$$\frac{\partial Y_i}{\partial K_i} = \alpha\theta \left(\frac{g}{k_i}\right)^{1-\alpha} = r \tag{1}$$

$$\frac{\partial Y_i}{\partial L_i} = (1-\alpha)\,\theta k_i^{\alpha} g^{1-\alpha} = w, \tag{2}$$

where $k_i = K_i/L_i$ is the firm-i capital-labor ratio, r is the market (gross) return to capital, and w is the market (gross) wage. As all firms are equal, they choose the same capital-labor ratio, $k_i \equiv K/L_2$, where K denotes aggregate capital; the production function can thus be aggregated:

$$Y = \theta L_2 \left(\frac{K}{L_2}\right)^{\alpha} g^{1-\alpha}, \tag{3}$$

which, in per worker terms (private plus public workers), becomes

$$y = \theta \frac{L_2}{L} \left(\frac{K}{L_2}\right)^{\alpha} g^{1-\alpha} = \theta (1-\lambda) \left(\frac{L}{L_2}\frac{K}{L}\right)^{\alpha} g^{1-\alpha} = \theta (1-\lambda)^{1-\alpha} k^{\alpha} g^{1-\alpha} \tag{4}$$

where $y = Y/L$, $k = K/L$, and $(1-\lambda) = L_2/L$.

In equilibrium, the net return on capital, $\bar{r}$, is equal to $\bar{r} = (1-\tau)r$, where r is given by (1) and, in view of (4), can be rewritten in per worker terms as

$$r = \frac{\partial y}{\partial k} = \alpha\theta (1-\lambda)^{1-\alpha} k^{\alpha-1} g^{1-\alpha} = \alpha \frac{y}{k}, \tag{5}$$

while the net wage of a private employee (and public official) is equal to $\bar{w} = (1-\tau)w$, where w is given by (2) and, in view of (4), can be rewritten in per worker terms as

$$w = \frac{\partial y}{\partial (1-\lambda)} = (1-\alpha)\theta (1-\lambda)^{-\alpha} k^{\alpha} g^{1-\alpha} = (1-\alpha) \frac{y}{1-\lambda} \tag{6}$$

so that the per worker gross private output is given by $rk + w(1-\lambda) = y$.

It's worth noticing that if y and k grow at the same rate, then r is constant over time and also the wage w will grow at the output (capital) rate.

2.2 Government and the Static General Equilibrium

Government employs $L_1 = L - L_2$ public officials to monitor the public expenditures for intermediate goods and services used by the private firms as input of their production process. Public official's wage is the same of the private worker's one and it is paid by the Government using taxes. We assume Government has a balanced budget. Total tax revenues are equal to $T = \tau\tilde{Y}$ where $\tilde{Y} = Y + L_1 w$ is the total taxable national income. The public administration spends such amount in intermediate goods and services to private firms and in public officials' wages, that is,

$$T = \tau\tilde{Y} = \tau Y + \tau L_1 w = \tilde{G} + L_1 w,$$

where $\tilde{G}$ denotes the *potential* amount of resources to be devoted to the firms as intermediate goods and services, which is given by

$$\tilde{G} = \tau Y + \tau L_1 w - L_1 w = \tau Y - (1-\tau) L_1 w. \tag{7}$$

However, intermediate goods and services that actually reach firms are not $\tilde{G}$ but $G = [1 - \mathbb{E}(s)]\tilde{G}$, as public officials will steal on average the share $\mathbb{E}(s)$ of public resources committed to that scope. Moreover, taking into account the effect of congestion, only the share

$$g = \frac{G}{L} = \frac{[1-\mathbb{E}(s)]\tilde{G}}{L} = [1-\mathbb{E}(s)][\tau y - (1-\tau)\lambda w] \tag{8}$$

will eventually enter the production function of each firm as input.

Substituting g as in (8) into (4) and using (6) yields

$$\begin{aligned}
y &= \theta\left(1-\lambda\right)^{1-\alpha}k^{\alpha}g^{1-\alpha} = \theta\left(1-\lambda\right)^{1-\alpha}k^{\alpha}\left[1-\mathbb{E}\left(s\right)\right]^{1-\alpha}\left[\tau y-\left(1-\tau\right)\lambda w\right]^{1-\alpha} \\
&= \theta\left(1-\lambda\right)^{1-\alpha}k^{\alpha}\left[1-\mathbb{E}\left(s\right)\right]^{1-\alpha}\left[\tau y-\left(1-\tau\right)\lambda\left(1-\alpha\right)\frac{y}{1-\lambda}\right]^{1-\alpha} \\
&= \theta\left(1-\lambda\right)^{1-\alpha}k^{\alpha}y^{1-\alpha}\left[1-\mathbb{E}\left(s\right)\right]^{1-\alpha}\left[\frac{\tau\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda}{1-\lambda}\right]^{1-\alpha} \\
&= \theta k^{\alpha}y^{1-\alpha}\left[1-\mathbb{E}\left(s\right)\right]^{1-\alpha}\left[\tau\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda\right]^{1-\alpha},
\end{aligned}$$

from which it turns out that per worker private output is a linear function of per worker capital:

$$y = \theta^{\frac{1}{\alpha}}\left[1-\mathbb{E}\left(s\right)\right]^{\frac{1-\alpha}{\alpha}}\left[\tau\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda\right]^{\frac{1-\alpha}{\alpha}}k, \tag{9}$$

that is, our economy resembles the features of a typical 'AK' model. In order to be defined, the RHS of (9) requires the following assumption.

A. 1 *Parameters* α, λ *and* τ *must satisfy* $\tau\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda > 0$, *which may be conveniently rewritten as* $\left(1-\alpha\lambda\right)\tau-\left(1-\alpha\right)\lambda > 0$, *that is, the following condition must hold:*

$$\tau > \frac{\left(1-\alpha\right)\lambda}{1-\alpha\lambda}.$$

Substituting y as in (9) into (8) and using again (6) shows that g turns out to be linear in k as well:

$$\begin{aligned}
g &= \left[1-\mathbb{E}\left(s\right)\right]\left[\tau y-\left(1-\tau\right)\lambda\left(1-\alpha\right)\frac{y}{1-\lambda}\right] \\
&= \left[1-\mathbb{E}\left(s\right)\right]\frac{\tau\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda}{1-\lambda}y \\
&= \theta^{\frac{1}{\alpha}}\left[1-\mathbb{E}\left(s\right)\right]^{\frac{1}{\alpha}}\left[\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda\right]^{\frac{1}{\alpha}}\left(1-\lambda\right)^{-1}k.
\end{aligned} \tag{10}$$

Similarly, from (5) and (9) it is immediately seen that the interest rate is given by

$$r = \alpha\frac{y}{k} = \alpha\theta^{\frac{1}{\alpha}}\left[1-\mathbb{E}\left(s\right)\right]^{\frac{1-\alpha}{\alpha}}\left[\tau\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda\right]^{\frac{1-\alpha}{\alpha}}, \tag{11}$$

while, from (6) and (9) it is easily seen that the gross market wage is given by

$$w = \frac{\left(1-\alpha\right)y}{1-\lambda} = \left(1-\alpha\right)\theta^{\frac{1}{\alpha}}\left[1-\mathbb{E}\left(s\right)\right]^{\frac{1-\alpha}{\alpha}}\frac{\left[\tau\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda\right]^{\frac{1-\alpha}{\alpha}}}{1-\lambda}k. \tag{12}$$

Note that, if the average share of public resources stolen by public officials, $\mathbb{E}\left(s\right)$, is constant, then the interest rate r in (11) and the gross wage in (12) turn out to be constant and a linear function of per worker capital respectively. We shall see in the next sections that this is actually the case.

Let us denote by $\tilde{g} = \tilde{G}/L$ the per worker supply of intermediate goods and services potentially available to firms, and by q the amount of public resources under the control of each public official that enter her intertemporal budget constraint, *i.e.*, before the public official takes a decision on what portion of it she is ready to steal. Then, by (7), (6), and (9),

$$\begin{aligned}
q &= \frac{\tilde{G}}{L_1} = \frac{\tilde{g}}{\lambda} = \frac{\tau y}{\lambda}-\left(1-\tau\right)w = \left[\frac{\tau}{\lambda}-\frac{\left(1-\tau\right)\left(1-\alpha\right)}{1-\lambda}\right]y \\
&= \theta^{\frac{1}{\alpha}}\left[1-\mathbb{E}\left(s\right)\right]^{\frac{1-\alpha}{\alpha}}\frac{\left[\tau\left(1-\lambda\right)-\left(1-\tau\right)\left(1-\alpha\right)\lambda\right]^{\frac{1}{\alpha}}}{\lambda\left(1-\lambda\right)}k
\end{aligned} \tag{13}$$

Clearly, as y, g, w and q are all linear functions of per worker capital, k, if $\mathbb{E}(s)$ is constant through time they all will grow at the same rate. Moreover, note that the ratio w/q is always a constant:

$$\frac{w}{q} = \frac{(1-\alpha)\lambda}{\tau(1-\lambda) - (1-\tau)(1-\alpha)\lambda} \tag{14}$$

2.3 Private Employees

At each given time t all young private employees in the t-cohort solve the same deterministic two-period maximization problem:[3]

$$\max_{\{x_t\}} \left(\ln c_{1,t} + \beta \ln c_{2,t+1}\right) \tag{15}$$

$$\text{s.t.} \begin{cases} c_{1,t} = \bar{w}_t - x_t \\ c_{2,t+1} = (1+\bar{r}_{t+1})\, x_t, \end{cases} \tag{16}$$

where $c_{1,t}$ and $c_{2,t+1}$ denote consumption in the first and second period respectively, x_t denotes the asset amount (saving) to be chosen, while $\bar{r}_{t+1} > 0$ is the net of taxes interest rate, and $\bar{w}_t > 0$ is the net of taxes wage earned. They are defined as $\bar{r} = (1-\tau)\, r$ and $\bar{w} = (1-\tau)\, w$, where the the tax rate $0 < \tau < 1$, as well as the gross interest rate r and gross wage w in (11) and (12) respectively, are taken as exogenously given.

After replacing $c_{1,t}$ and $c_{2,t+1}$ according to the constraints (16) into the objective function (15), the FOC with respect to the asset x_t yields the optimal individual saving as a fraction of the wage $\bar{w}_t$:

$$x_t = \frac{\beta}{1+\beta}\bar{w}_t = \frac{\beta(1-\tau)}{1+\beta} w_t. \tag{17}$$

It is well known that the "canonical" OLG model with logarithmic utility yields an optimal saving amount which is independent of the (net) interest rate $\bar{r}_{t+1}$ (see Section 9.3 in Acemoglu, 2009).

2.4 Public Officials

Unlike private workers, each public official has the opportunity to divert a fraction $0 \leq s \leq 1$ of the amount q of public resources under her own control as given in (13), and add such amount to their individual asset when she is young at time t. During the same initial period in her life, but after she took her optimal decision on how much to steal, she may get caught by the authorities, in which case she must give back the whole amount stolen and pay a fine $\varphi > 0$ per unit of resource stolen. The probability of being caught is $0 < p < 1$, constant through time.

At each given time t all young public officials solve the same stochastic two-period maximization problem:[4]

$$\max_{\{x_t, s_t\}} \mathbb{E}\left(\ln c_{1,t} + \beta \ln c_{2,t+1}\right) \tag{18}$$

$$\text{s.t.} \begin{cases} c_{1,t} = \bar{w}_t - x_t + (1-z_t)\, q_t s_t - z_t f q_t s_t \\ c_{2,t+1} = (1+\bar{r}_{t+1})\, x_t \\ 0 \leq s_t < 1, \end{cases} \tag{19}$$

where $\mathbb{E}$ denotes time t expectation, $c_{1,t}$ and $c_{2,t+1}$ denote consumption in the first and second period respectively, x_t denotes the asset amount (saving) to be chosen, s_t the share of public resources under

[3]As all individuals are the same, we drop the index i indicating each of them.

[4]As all individuals are the same, we drop the index j indicating each of them.

control, q_t, that will be stolen at time t, $f = 1 + \varphi > 1$ is the amount that must be returned to the Government in the event of being caught, while again $\bar{r}_{t+1} > 0$ is the net of taxes interest rate, and $\bar{w}_t > 0$ is the net of taxes wage earned. The latter are the same as those of private employees and are defined as $\bar{r} = (1 - \tau) r$ and $\bar{w} = (1 - \tau) w$.

The indicator function z_t is associated to probability p of being caught at time t is defined as

$$z_t = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1 - p, \end{cases} \tag{20}$$

and it is unknown (*i.e.*, it is a random variable) at the time in which the (optimal) decision is taken upon x_t and s_t, but it is revealed before the instant in which the amount $c_{1,t}$ is being consumed; therefore, the first constraint in (19) is truly random but affects only the consumption $c_{1,t}$ of young public officials, as the consumption $c_{2,t+1}$ in the old age is fully determined by the interest rate $\bar{r}_{t+1}$, which is deterministic and exogenously given, and by the choice on savings x_t, which has already been taken. $\{z_t\}_{t=0}^{\infty}$ is a process of i.i.d. Bernoulli random variables such that $\Pr(z_t = 1) = p$, where $0 < p < 1$ corresponds to the probability that each public official will be caught to steal in the period between her optimal decisions and her consumption. In other words, the amount of consumption in the old age, $c_{2,t+1}$, is not being affected by the realization of the random variable z_t one period before (in the young age), as the decision on the optimal saving x_t has been taken before the administration controls take place, and cannot be modified. We admit that this is a quite strong assumption, but, if on one hand it is useful to simplify the analysis, on the other hand we consider unrealistic that each public official must wait until retirement to know whether she has being caught or she can get it free.

We assume that q_t is exogenously given according to (13) and that public officials maximize their total expected utility independently from each other. Moreover, the Bernoulli process is assumed to be i.i.d. both over time and across public officials.

After replacing $c_{1,t}$ and $c_{2,t+1}$ according to the constraints (19) into the objective function (18), the problem can be rewritten as

$$\begin{gathered} \max_{\{x_t, s_t\}} \left\{ (1-p) \ln (\bar{w}_t - x_t + q_t s_t) + p \ln (\bar{w}_t - x_t - f q_t s_t) \right. \\ \left. + \beta \ln \left[(1 + \bar{r}_{t+1}) x_t \right] \right\} \\ \text{s.t. } 0 \le s_t \le 1. \end{gathered} \tag{21}$$

Assuming an interior solution, $x_t > 0$, $0 < s_t < 1$, FOC on (21) yield the following optimal individual saving, which turns out to be the same as that in (17) for private workers:

$$x_t = \frac{\beta}{1+\beta} \bar{w}_t = \frac{\beta (1-\tau)}{1+\beta} w_t, \tag{22}$$

while the optimal individual stealing choice s_t turns out to be a fraction of the ratio $\bar{w}_t / q_t$:

$$s_t = \frac{1 - p(f+1)}{(1+\beta) f} \frac{\bar{w}_t}{q_t} = \frac{[1 - p(f+1)](1-\tau)}{(1+\beta) f} \frac{w_t}{q_t}. \tag{23}$$

Recall from (14) that the ratio between the (exogenous) wage in (12) and the amount of public resources under the control of each public official in (13) is constant. Hence, as public officials are all equal and s_t in (23) depends only on parameters β, τ, p, f plus the exogenous variables w_t and q_t, we have just established the next result, that will be crucial in the following analysis

Proposition 1 *Under Assumption A.1, if*

$$p < \frac{1}{1+f} \quad \textit{and} \quad \tau > \frac{[1 - p(f+1) + (1+\beta) f](1-\alpha)\lambda}{[1 - p(f+1)](1-\alpha)\lambda + (1+\beta) f (1 - \alpha\lambda)}, \tag{24}$$

then the following hold.

i) *At each time t, all public officials steal the same amount $s_t \equiv s$ constant through time, which, according to (23) and (14) is given by*

$$s = \frac{[1 - p(f+1)](1-\tau)(1-\alpha)\lambda}{(1+\beta) f [\tau(1-\lambda) - (1-\tau)(1-\alpha)\lambda]}. \quad (25)$$

ii) *Therefore, also the average theft is constant through time, $\mathbb{E}(s_t) \equiv s$, and y_t, g_t, w_t, q_t, being all linear functions of per worker capital, k_t, grow at the same constant rate whenever the economy features sustained growth.*

iii) *The optimal theft s defined in (25) is decreasing in p, f and τ.*

3 Aggregate Equilibrium and Growth

Under the assumption that all agents have logarithmic utility, the optimal savings x_t of everybody, either private worker or public official, are the same and are given by (22). Assuming that savings of young agents at time t are employed as physical capital in time $t+1$ by private firms [see eqn. (3.105) in Barro and Sala-i-Martin, 2004], we can exploit the linear, 'AK', structure of the production process in our economy discussed at the end of Section 2.2 to immediately compute the BGP growth rate:

$$\begin{aligned} k_{t+1} = x_t &= \frac{\beta(1-\tau)}{1+\beta} w_t \\ &= \frac{\beta(1-\tau)(1-\alpha)}{1+\beta} \theta^{\frac{1}{\alpha}} [1 - \mathbb{E}(s)]^{\frac{1-\alpha}{\alpha}} [\tau(1-\lambda) - (1-\tau)(1-\alpha)\lambda]^{\frac{1-\alpha}{\alpha}} (1-\lambda)^{-1} k_t \\ &= \frac{\beta(1-\tau)(1-\alpha)}{1+\beta} \theta^{\frac{1}{\alpha}} (1-s)^{\frac{1-\alpha}{\alpha}} [\tau(1-\lambda) - (1-\tau)(1-\alpha)\lambda]^{\frac{1-\alpha}{\alpha}} (1-\lambda)^{-1} k_t \\ &= \Psi(\tau, \lambda) k_t, \end{aligned} \quad (26)$$

where in the second equality we used (22), in the third (12), in the fourth condition (25) of Proposition 1 establishing that $\mathbb{E}(s)$ is constant, $\mathbb{E}(s) \equiv s$, while in the last equality we emphasize the dependence on parameters τ and λ of the constant Ψ, defined as

$$\begin{aligned} \Psi(\tau, \lambda) = \; & \frac{\beta(1-\alpha)\theta^{\frac{1}{\alpha}}(1-\tau)}{(1+\beta)(1-\lambda)} \\ & \times \left\{ \tau(1-\lambda) - \left[1 + \frac{1 - p(f+1)}{(1+\beta) f}\right](1-\tau)(1-\alpha)\lambda \right\}^{\frac{1-\alpha}{\alpha}}, \end{aligned} \quad (27)$$

because in the sequel we will focus on comparative dynamics based on the tax rate τ and the share of workers employed in the public sector, $\lambda = L_1/L$, parameters that can be both interpreted as proxies of "Government size". Note that the term in curly brackets is certainly positive because it is the result of the product $(1-s)[\tau(1-\lambda) - (1-\tau)(1-\alpha)\lambda]$ under the same exponent, $(1-\alpha)/\alpha$, where $[\tau(1-\lambda) - (1-\tau)(1-\alpha)\lambda] > 0$ under Assumption A.1 and $(1-s) > 0$ under condition (24) of Proposition 1.

The linear difference equation of per worker physical capital defined in (26) immediately yields the positive growth rate of the economy,

$$\gamma(\tau, \lambda) = \Psi(\tau, \lambda) - 1, \quad (28)$$

provided that parameters α, β, θ, λ, τ, f and p are such that $\Psi(\tau, \lambda) > 1$. The growth rate $\gamma(\tau, \lambda)$ in (28) is constant an thus characterizes the only possible BGP on which the economy jumps immediately starting from $t = 0$.

Proposition 2 *Under Assumption A.1 and the assumptions of Proposition 1 the following hold.*

i) *For any fixed admissible pair* (τ, λ) *the economy growth rate* $\gamma(\tau, \lambda)$ *defined in (28) is an increasing function of both the probability of detection* p *and the fine* f.

ii) *For any fixed* $0 < \lambda < 1$, *the economy growth rate* $\gamma(\cdot, \lambda)$ *defined in (28) is an inverted U-shaped function of the tax rate* τ *and admits one unique interior maximum point,* $0 < \tau^*(\lambda) < 1$, *which is itself function of all parameters according to*

$$\tau^*(\lambda) = \frac{[1 - p(f + 1)](1 - \alpha)\lambda + (1 + \beta) f (1 - \alpha)}{[1 - p(f + 1)](1 - \alpha)\lambda + (1 + \beta) f (1 - \alpha\lambda)}. \tag{29}$$

iii) *Under the following further restrictions:*

$$\alpha > \left(\sqrt{5} - 1\right)/2 \simeq 0.618 \quad \textit{and} \quad \frac{1 - p(f + 1)}{(1 + \beta) f} < \frac{\alpha^2 + \alpha - 1}{\alpha(1 - \alpha)} \tag{30}$$

the growth rate $\gamma(\tau, \lambda)$ *defined in (28), considered as a function of both parameters* τ *and* λ, *admits one unique (interior) stationary point* (τ^*, λ^*) *with coordinates*

$$\tau^* = \frac{(2\alpha - 1)(1 + \beta) f - [1 - p(f + 1)](1 - \alpha)}{\alpha(1 + \beta) f - [1 - p(f + 1)](1 - \alpha)} \tag{31}$$

$$\lambda^* = \frac{(\alpha^2 + \alpha - 1)(1 + \beta) f - [1 - p(f + 1)]\alpha(1 - \alpha)}{(2\alpha - 1)\{\alpha(1 + \beta) f - [1 - p(f + 1)](1 - \alpha)\}}. \tag{32}$$

iv) *Both stationary values* τ^* *and* λ^* *in (31) and (32) are increasing functions of both the probability of detection* p *and the fine* f.

Clearly, assumptions (30) are necessary to have a positive numerator in the expression for λ^* in (32); in the proof it is shown that the same conditions imply that the denominator is positive as well. Note that the second condition in (30) holds if f and p are large enough, provided that they satisfy the first condition in (24) of Proposition 1.

Unfortunately, it is not possible to establish concavity of $\Psi(\tau, \lambda)$, and thus of $\gamma(\tau, \lambda) = \Psi(\tau, \lambda) - 1$, directly; however, graphic inspection shows that, for reasonable values of parameters in their admissible ranges, it should be [see, *e.g.*, Figure 1(a) in the next Section]. Nonetheless, we do not really need to know that the unique stationary point (τ^*, λ^*) with coordinates given by (31) and (32) is a global maximum point: assuming that the economy adopts the (optimal) tax rate τ^* as in (31) and the Government employs a share λ^* as in (32) of total labour, part (iv) of Proposition 2 establishes that, if parameter p or f, or both, increase, then there exist a direction originating from (τ^*, λ^*) along which, in order to keep the growth rate $\gamma(\tau, \lambda) = \Psi(\tau, \lambda) - 1$ in (28) at its maximum level, an increase of the optimal tax rate τ^* is required, provided that also the share λ^* is adjusted (increased) as well according to (32). In other words, although we are not able to prove that for different values of p and f conditions (31) and (32) describe the upper envelope of the function $\gamma(\tau, \lambda) = \Psi(\tau, \lambda) - 1$ (but we conjecture they do, as it will be illustrated through an example in the next section), part (iv) of Proposition 2 provides a criterion to follow (by adjusting the share of public employees, λ^*) in order to keep the (positive) monotonicity that links the optimal tax rate τ^* to increases in the Social Capital—represented by increases in either p or f, or both—that is found in empirical data. As both parameters p and f can be considered as proxy measures of the level of Social Capital, we have thus shown that an increase of Social Capital requires larger levels of both the tax rate τ^* and the share of public employees λ^* in order to keep the growth rate $\gamma(\tau^*, \lambda^*) = \Psi(\tau^*, \lambda^*) - 1$ at its maximum.

4 A Numerical Example

To carry out a numerical example we set the following parameters' values:

$$\beta = 0.3, \qquad \alpha = 0.67, \qquad \theta = 10.6.$$

The individual discount rate value is compatible with some 30 years time-horizon for a generation to be employed either in the private or in the public sector, the physical capital factor share value clearly satisfies the first condition in (30), and the technology parameter value guarantees reasonable values of the growth rate defined in (28) around its maximum points, as it will be shown below.

Note that the second condition in (30) of Proposition 2 can be reformulated as a lower bound for the probability of detection p given the fine f according to:

$$p > \frac{1}{1+f}\left[1 - \frac{(\alpha^2 + \alpha - 1)(1+\beta) f}{\alpha(1-\alpha)}\right].$$

Joining this condition with the first condition in (24) of Proposition 1 yields the following open interval as feasible range for the probability of detection p for any given value of the fine f:

$$p \in \left(\frac{1}{1+f}\left[1 - \frac{(\alpha^2 + \alpha - 1)(1+\beta) f}{\alpha(1-\alpha)}\right], \frac{1}{1+f}\right),$$

whose endpoints, when considered as functions of the fine f for given α and β values, are two hyperbolas, the former laying strictly below the latter whenever $f > 0$, while they collapse into the same point $p = 1$ when $f = 0$. The right endpoint is strictly positive for any $f > 0$, while the left endpoint intersects the horizontal axis on the point

$$f_0 = \frac{\alpha(1-\alpha)}{(\alpha^2 + \alpha - 1)(1+\beta)},$$

that is, on the value $f_0 = 1.43$ when $\alpha = 0.67$ and $\beta = 0.3$.

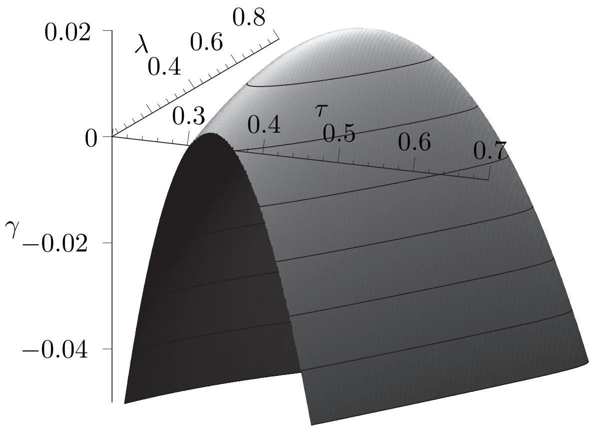
We focus on how changes in the probability of detection p affect the growth rate function $\gamma(\tau, \lambda) = \Psi(\tau, \lambda) - 1$ defined in (28), paying special attention to its maximum point $\gamma(\tau^*, \lambda^*)$ with coordinates τ^* and λ^* given by (31) and (32). To this purpose, we fix the value of the fine at $f = f_0 = 1.43$, so to have the largest possible range for the p values, given by the interval $p \in (0, 0.41)$. Similar results are obtained for increasing values of the fine f for a fixed probability p, or for increasing values of both f and p; we omit these types of illustration. Under these assumptions—*i.e.*, for $\beta = 0.3$, $\alpha = 0.67$, $\theta = 10.6$ and $f_0 = 1.43$ fixed—we consider seven values for the probability of detection p in the range $(0, 0.41)$ and compute the coordinates τ^* and λ^* according to (31) and (32), plus the corresponding maximum growth rate value $\gamma(\tau^*, \lambda^*)$, for each probability value considered. The results are reported in Table 1, where the monotonic increasing pattern of both the optimal tax rate τ^* and optimal share of public employees λ^*, as well as the corresponding maximum value of the growth rate $\gamma(\tau^*, \lambda^*)$, predicted by Proposition 2 is clearly evident as the probability of detection p increases.

Figure 1(a) shows the three-dimensional graph of the growth rate $\gamma(\tau, \lambda)$ defined in (28) as a function of τ and λ for $f_0 = 1.43$ and $p = 0.20$: it clearly exhibits 'concavity' traits, with, according to the fourth row of Table 1, a unique global interior maximum point reached on the pair $(\tau^*, \lambda^*) = (0.4299, 0.2937)$ and with value $\gamma(\tau^*, \lambda^*) = 0.0173$, corresponding to an optimal growth rate of around 1.7%. Figure 1(b) presents an attempt to draw an imaginary upper envelope curve in the three-dimensional space (τ, λ, γ) of the seven maximum points defined by the three values listed in the last three columns of Table 1 by ideally joining the seven red dots in the figure, each corresponding to the maximum growth rate value for the graph of $\gamma(\tau, \lambda)$ determined by the seven values of the probability

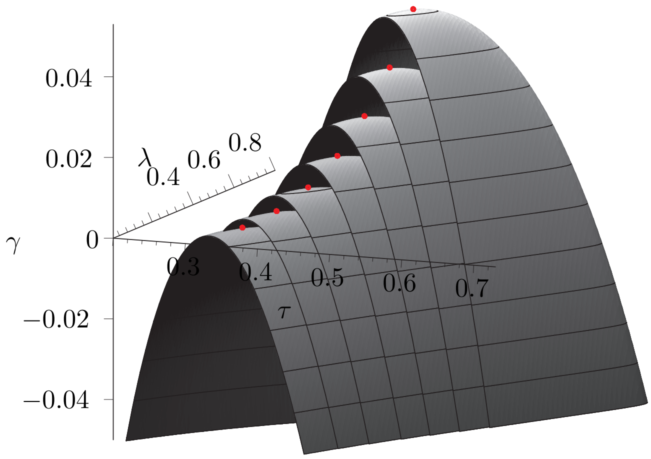
p	τ^*	λ^*	$\gamma(\tau^*, \lambda^*)$
0.05	0.3581	0.0827	0.0031
0.10	0.3839	0.1587	0.0059
0.15	0.4078	0.2288	0.0106
0.20	0.4299	0.2937	0.0173
0.25	0.4503	0.3539	0.0262
0.30	0.4694	0.4100	0.0373
0.35	0.4872	0.4623	0.0509

TABLE 1: optimal values for the tax rate τ^* and the public employees share λ^* according to (31) and (32), and the corresponding maximum growth rate $\gamma(\tau^*, \lambda^*)$ according to (28), for seven feasible values of the probability of detection p; $\beta = 0.3$, $\alpha = 0.67$, $\theta = 10.6$ and $f = f_0 = 1.43$.

of detection p in the first column of Table 1. Note that, consistently with part (i) of Proposition 2, each graph of the growth rate function $\gamma(\tau, \lambda)$ plotted in Figure 1(b) is a surface (only partially reported for $p \geq 0.10$ so to emphasize their area close to their maximum points) that lays uniformly above the other surfaces corresponding to lower values of p and uniformly below the other surfaces corresponding to higher values of p. This feature renders difficult a three-dimensional graphical representation of the upper envelope of all the functions $\gamma(\tau, \lambda)$ as p increases and justifies the choice of plotting only partial sections of the surfaces [the graphs of the function $\gamma(\tau, \lambda)$] corresponding to probability values larger than 0.10.



(a)



(b)

FIGURE 1: (a) three-dimensional plot of the growth rate function $\gamma(\tau, \lambda) = \Psi(\tau, \lambda) - 1$ defined in (28) for $f = f_0 = 1.43$ and $p = 0.20$; (b) several three-dimensional plots of the same growth rate function—each uniformly higher then the other—for $f = f_0 = 1.43$ and the seven values of p listed in the first column of Table 1, the red dots denote the maximum value for each function.

While the increasing pattern of the single three-dimensional graphs of $\gamma(\tau, \lambda)$—further emphasized by their maximum points denoted by red dots, which provide a skeleton for drawing their upper envelope—clearly confirms part (i) of Proposition 2, the monotonic increasing pattern of both coordinates τ^* and λ^* established in part (iv) of Proposition 2 as p increases (our main result) can only be inferred from Figure 1(b). Figure 2 provides a better flavour of part (iv) of Proposition 2 by plotting separately the λ^* and τ^*-sections respectively of the growth rate function $\gamma(\tau, \lambda)$ for the seven p values in the fist column of Table 1: specifically, Figure 2(a) plots the two-dimensional graphs of

$\gamma(\tau, \lambda^*)$ as a function of the only variable τ for each optimal value λ^* reported in the third column of Table 1, while Figure 2(b) reports the two-dimensional graphs of $\gamma(\tau^*, \lambda)$ as a function of the only variable λ for each optimal value τ^* reported in the second column of Table 1. The increasing pattern of the maximum points, as well as the maximum values, of each curve as p increases is apparent in both figures. However, from Figure 1(b) we learn that each curve should be projected deeper and deeper in the third dimension orthogonal to the (τ, γ) and (λ, γ) spaces respectively as λ^* and τ^* increase for larger values of p; that is, each curve in Figure 2(a) corresponds to larger values of λ^*, while each curve in Figure 2(b) corresponds to larger values of τ^*, so that the upper envelope Figure 1(b) develops (increases) toward north-east in the (τ, λ) space.

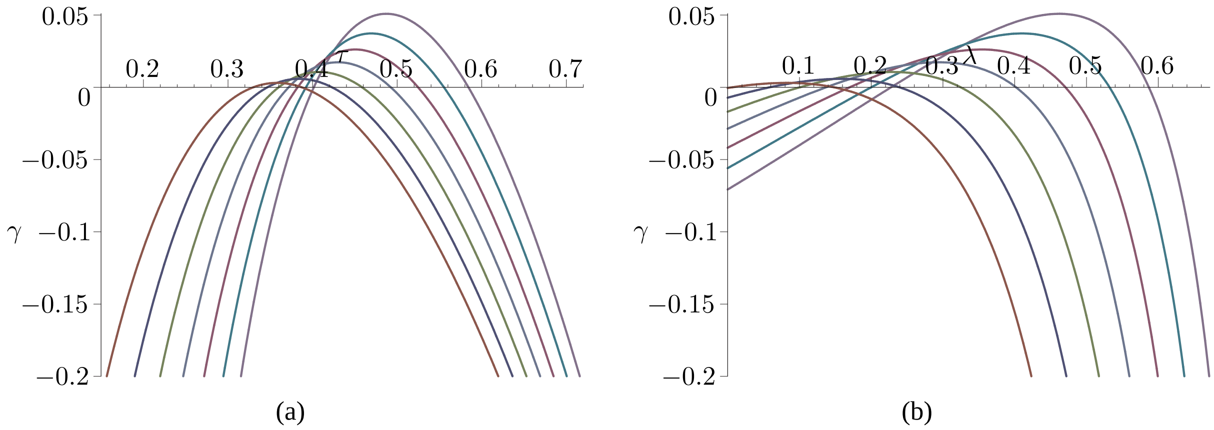


FIGURE 2: (a) two-dimensional plots of the λ^*-sections of the growth rate function $\gamma(\tau, \lambda) = \Psi(\tau, \lambda) - 1$ defined in (28) for $f = f_0 = 1.43$ and the seven values of the pairs p, λ^* listed in the first and third columns of Table 1; (b) two-dimensional plots of the τ^*-sections of the same growth rate function for $f = f_0 = 1.43$ and the seven values of the pairs p, τ^* listed in the first and second columns of Table 1.

5 Conclusions

Using the OLG framework, we have shown that a simple modification of the Barro (1990)'s endogenous growth model, introduced to take into account the possibility that public officials will steal a fraction of public resources under their own control, is capable of theoretically explain the existence of high growth rates in the presence of Big Government size. Specifically, we have shown that, under realistic conditions on parameters' values, a monotonic increasing relationship exists among the level of Social Capital (expressed in terms of either the probability to detect cheating public officials or the fine charged to them), the Government size (expressed both in terms of the tax rate and the number of public employees) and the maximum achievable economic growth rate. Social Capital could thus be the missing dimension accounting for the controversial empirical results on this issue as well as for the ENC case. High levels of Social Capital affect the behavior of public officials monitoring the public expenditures for intermediate goods and services supplied to private firms, and thus the efficiency of Government as a whole.

Appendix

Proof of Proposition 1. To prove i) we only must show that the optimal theft s in (25) is interior, that is it satisfies $0 < s < 1$, and that the argument of the utility in the second term in (21) is strictly positive, $\bar{w}_t - x_t - fq_t s_t > 0$, when x_t and s_t are defined according to (22) and (23) respectively, that is, the optimal choice on x_t and s_t must yield positive consumption in the unlucky event of being caught.

The former property is a consequence of Assumption A.1 together with condition (24). First note that Assumption A.1 requires the term $\tau(1-\lambda) - (1-\tau)(1-\alpha)\lambda$ in the denominator of (25) to be positive, so that, in order to have $s > 0$ in (25), as the term $(1-\tau)(1-\alpha)\lambda$ is positive by construction, the term $1 - p(f+1)$ in the numerator must be positive as well, that is, the first condition in (24) must hold. To have $s < 1$ as well we solve

$$\frac{[1-p(f+1)](1-\tau)(1-\alpha)\lambda}{(1+\beta)f[\tau(1-\lambda)-(1-\tau)(1-\alpha)\lambda]} < 1,$$

which, under Assumption A.1, is equivalent to

$$\{[1-p(f+1)](1-\alpha)\lambda + (1+\beta)f(1-\alpha\lambda)\}\tau > [1-p(f+1)+(1+\beta)f](1-\alpha)\lambda,$$

which immediately yields the second condition in (24). It is easily shown that the second condition in (24) is stronger than Assumption A.1 on τ as, using the fact that the first condition in (24) is equivalent to $1-p(f+1) > 0$, the inequality

$$\frac{[1-p(f+1)+(1+\beta)f](1-\alpha)\lambda}{[1-p(f+1)](1-\alpha)\lambda+(1+\beta)f(1-\alpha\lambda)} > \frac{(1-\alpha)\lambda}{1-\alpha\lambda}$$

boils down to $\lambda < 1$, which holds by construction.

The latter property follows directly from (22) and (23):

$$\bar{w}_t - x_t - fq_t s_t = \bar{w}_t - \frac{\beta}{1+\beta}\bar{w}_t - fq_t\frac{1-p(f+1)}{(1+\beta)f}\frac{\bar{w}_t}{q_t} = \frac{p(f+1)}{1+\beta}\bar{w}_t,$$

where the last term is positive whenever $\bar{w}_t = (1-\tau)w_t > 0$.

Property ii) is an immediate consequence of s in (25) being constant and all the discussion in Subsection 2.2.

Finally, to prove iii), direct computation of $\partial s/\partial p$, $\partial s/\partial f$ and $\partial s/\partial \tau$ in (25) show that they are all negative under our assumptions on all parameters. ■

Proof of Proposition 2. Property i) is established by direct computation of the partial derivatives with respect to p and f of the function $\gamma(\tau,\lambda) = \Psi(\tau,\lambda) - 1$ defined in (28), which turn out to be both positive.

To prove ii) let

$$A = \frac{\beta(1-\alpha)\theta^{\frac{1}{\alpha}}}{(1+\beta)} \quad \text{and} \quad B = \left[1 + \frac{1-p(f+1)}{(1+\beta)f}\right](1-\alpha) \tag{33}$$

so that we can rewrite $\Psi(\tau,\lambda)$ in (27) as

$$\Psi(\tau,\lambda) = A\left(\frac{1-\tau}{1-\lambda}\right)[\tau(1-\lambda)-(1-\tau)B\lambda]^{\frac{1-\alpha}{\alpha}}. \tag{34}$$

Note that under all our assumptions, including the first condition in (24) which implies that $1-p(f+1) > 0$, the constants in (33) are positive: $A, B > 0$. The term in square brackets on the RHS of (34) is positive because it is the result of the product $(1-s)[\tau(1-\lambda)-(1-\tau)(1-\alpha)\lambda]$ with $[\tau(1-\lambda)-(1-\tau)(1-\alpha)\lambda]>0$ under Assumption A.1 and $(1-s)>0$ under condition (24) of Proposition 1. Specifically, we have:

$$\tau(1-\lambda)-(1-\tau)B\lambda>0 \quad \text{for all } 0<\lambda<1 \quad \text{and} \quad \tau_L(\lambda)<\tau<1,$$

where $\tau_L(\lambda)$ is the lower bound in the admissible range for τ defined by the second condition in (24),

$$\tau_L(\lambda)=\frac{[1-p(f+1)+(1+\beta)f](1-\alpha)\lambda}{[1-p(f+1)](1-\alpha)\lambda+(1+\beta)f(1-\alpha\lambda)}, \tag{35}$$

where we stress its dependency on λ. For a given $0<\lambda<1$, the problem $\max\{\gamma(\tau)=\Psi(\tau)-1:\tau_L(\lambda)<\tau<1\}=\max\{\ln[\Psi(\tau)]:\tau_L(\lambda)<\tau<1\}$ can be written as

$$\max_{\tau_L(\lambda)<\tau<1}\left\{\ln A+\ln(1-\tau)-\ln(1-\lambda)+\frac{1-\alpha}{\alpha}\ln[\tau(1-\lambda)-(1-\tau)B\lambda]\right\}, \tag{36}$$

where we used (34). FOC on the RHS easily yields the optimal value

$$\tau^*(\lambda)=\frac{B\lambda+(1-\alpha)(1-\lambda)}{B\lambda+1-\lambda}. \tag{37}$$

Using the definition of B in (33), note that $[1-p(f+1)+(1+\beta)f](1-\alpha)=(1+\beta)fB$ and $[1-p(f+1)](1-\alpha)=(B+\alpha-1)(1+\beta)f$, so that, substituting into (35), we get the lower bound $\tau_L(\lambda)$ as a function of B and B as a function of $\tau_L(\lambda)$:

$$\tau_L(\lambda)=\frac{B\lambda}{B\lambda+1-\lambda}, \quad \text{and} \quad B=\frac{(1-\lambda)\tau_L(\lambda)}{\lambda[1-\tau_L(\lambda)]}. \tag{38}$$

Using the latter expression for B in in the expression (37) we can write the optimal tax rate as a function of the lower bound $\tau_L(\lambda)$ recalled in (35):

$$\tau^*(\lambda)=1-\alpha+\alpha\tau_L(\lambda), \tag{39}$$

from which, as $\tau_L(\lambda)<1$, it is apparent that $\tau^*(\lambda)>\tau_L(\lambda)$; while, as $\alpha<1$, from (37) it follows that $\tau^*(\lambda)<1$ as well, establishing that the unique stationary point $\tau^*(\lambda)$ is interior. Substituting $\tau_L(\lambda)$ as in (35) into (39),

$$\tau^*(\lambda)=1-\alpha+\frac{\alpha[1-p(f+1)+(1+\beta)f](1-\alpha)\lambda}{[1-p(f+1)](1-\alpha)\lambda+(1+\beta)f(1-\alpha\lambda)},$$

the expression in (29) is immediately obtained.

To confirm that $\tau^*(\lambda)$ is the unique solution of (36) for fixed λ, first note that, substituting B according to the second equation in (38) in the term $\tau(1-\lambda)-(1-\tau)B\lambda$, for $\tau>\tau_L(\lambda)$ it holds

$$\tau(1-\lambda)-(1-\tau)B\lambda=\frac{1-\lambda}{1-\tau_L(\lambda)}[\tau-\tau_L(\lambda)]>0,$$

moreover, using the first equation in (38) for $\tau_L(\lambda)$ in the term $\tau(1-\lambda)-(1-\tau)B\lambda$, for $\tau\to\tau_L^+(\lambda)$ one has

$$\tau(1-\lambda)-(1-\tau)B\lambda\to\frac{B\lambda(1-\lambda)-(1-\lambda)B\lambda}{B\lambda+1-\lambda}=0^+,$$

so that,

$$\lim_{\tau\to\tau_L^+(\lambda)}\frac{\partial}{\partial\tau}\ln\left[\Psi\left(\tau\right)\right]=\lim_{\tau\to\tau_L^+(\lambda)}\left\{-\frac{1}{1-\tau}+\frac{1-\alpha}{\alpha}\left[\frac{B\lambda+1-\lambda}{\tau\left(1-\lambda\right)-\left(1-\tau\right)B\lambda}\right]\right\}$$
$$=+\infty,$$

implying that $\left(\partial/\partial\tau\right)\ln\left[\Psi\left(\tau\right)\right]>0$ for all $\tau_L\left(\lambda\right)<\tau<\tau^*\left(\lambda\right)$. Next, it holds

$$\lim_{\tau\to1^-}\frac{\partial}{\partial\tau}\ln\left[\Psi\left(\tau\right)\right]=\frac{1-\alpha}{\alpha}\left(\frac{B\lambda+1-\lambda}{1-\lambda}\right)+\lim_{\tau\to1^-}\left(-\frac{1}{1-\tau}\right)=-\infty,$$

implying that $\left(\partial/\partial\tau\right)\ln\left[\Psi\left(\tau\right)\right]<0$ for all $\tau^*\left(\lambda\right)<\tau<1$. This establishes that $\ln\left[\Psi\left(\tau\right)\right]$ is inverted U-shaped functions and that $\tau^*\left(\lambda\right)$ in (29) is the unique solution of (36); as $\ln\left[\Psi\left(\tau\right)\right]$ is a monotone transformation of $\Psi\left(\tau\right)$, the same holds true for $\Psi\left(\tau\right)$, and in turn, for the growth rate $\gamma\left(\tau\right)=\Psi\left(\tau\right)-1$ defined in (28), for each fixed $0<\lambda<1$.

To establish iii) we consider the objective function in (36),

$$\begin{aligned}\ln\Psi\left(\tau,\lambda\right)=\ &\ln A+\ln\left(1-\tau\right)-\ln\left(1-\lambda\right)\\&+\frac{1-\alpha}{\alpha}\ln\left[\tau\left(1-\lambda\right)-\left(1-\tau\right)B\lambda\right],\end{aligned}\tag{40}$$

as a function of both variables τ and λ and study it over the open set $\left\{\left(\tau,\lambda\right):\left(0<\lambda<1\right)\wedge\left[\tau_L\left(\lambda\right)<\tau<1\right]\right\}$. As, according to (33), A and B do not depend on τ or λ, FOC with respect to λ on the RHS of (40) easily yields the critical value

$$\lambda^*=\frac{\left(1-2\alpha\right)\tau+\left(1-\alpha\right)\left(1-\tau\right)B}{\left(1-2\alpha\right)\left[\tau+\left(1-\tau\right)B\right]},$$

and pairing it with equation (37) leads to the unique stationary point with coordinates

$$\tau^*=\frac{\alpha-B}{1-B},\quad\text{and}\quad\lambda^*=\frac{2\alpha-1-\alpha B}{\left(2\alpha-1\right)\left(1-B\right)},\tag{41}$$

which, after replacing B according to (33) and through some algebra, establish (31) and (32). The two conditions in (30) clearly guarantee that $2\alpha-1-\alpha B>0\iff B<2-1/\alpha$, which, as $0<\left(1-\alpha\right)^2=\alpha^2-2\alpha+1\iff2-1/\alpha<\alpha$, in turn, implies $B<\alpha<1$; as $\alpha>\left(\sqrt{5}-1\right)/2>1/2\Longrightarrow\left(2\alpha-1\right)>0$, all these conditions together establish that both τ^* and λ^* are positive. Moreover, $\tau^*<1$ because $0<\alpha-B<1-B$, while $\lambda^*<1$ because $2\alpha-1-\alpha B<\left(2\alpha-1\right)\left(1-B\right)\iff\left(1-\alpha\right)B>0$. To check that $\tau^*>\tau_L\left(\lambda^*\right)$, with $\tau_L\left(\lambda^*\right)$ defined in (35) for $\lambda=\lambda^*$, we first use the expression of λ^* in (41) in the expression for B as in (38) to get

$$B=\frac{\left(1-\lambda^*\right)\tau_L\left(\lambda\right)}{\lambda^*\left[1-\tau_L\left(\lambda\right)\right]}=\frac{\left(1-\alpha\right)B\tau_L\left(\lambda\right)}{\left(2\alpha-1-\alpha B\right)\left[1-\tau_L\left(\lambda\right)\right]}\quad\iff\quad B=\frac{\alpha\left[2-\tau_L\left(\lambda\right)\right]-1}{\alpha\left[1-\tau_L\left(\lambda\right)\right]}.$$

Next, we replace the last expression for B in the first equation in (41) to get, after some algebra,

$$\tau^*=\frac{\alpha-B}{1-B}=\frac{1-\alpha\left(2-\alpha\right)}{1-\alpha}+\alpha\tau_L\left(\lambda\right);$$

as $\left[1-\alpha\left(2-\alpha\right)\right]/\left(1-\alpha\right)+\alpha\tau_L\left(\lambda\right)>\tau_L\left(\lambda\right)$ is equivalent to $1>\tau_L\left(\lambda\right)$, which is definitely true, we have shown that $\tau^*>\tau_L\left(\lambda\right)$. Therefore, we conclude that the stationary point $\left(\tau^*,\lambda^*\right)$ with coordinates given by (31) and (32) is an interior point of the open set $\left\{\left(\tau,\lambda\right):\left(0<\lambda<1\right)\wedge\left[\tau_L\left(\lambda\right)<\tau<1\right]\right\}$.

To prove iv) we first verify that both $\left(\partial/\partial p\right)B\left(p,f\right)$ and $\left(\partial/\partial f\right)B\left(p,f\right)$ are strictly negative through direct differentiation of B as in (33) with respect to p and f. Finally, using the fact that $\left(\partial/\partial p\right)B\left(p,f\right)<0$ and $\left(\partial/\partial f\right)B\left(p,f\right)<0$, by differentiating the expressions of both τ^* and λ^* as in (41) with respect to p and f it is easily established that that they are all strictly positive, and the proof is complete. ■

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