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UNIVERSITY DROPOUT PROBLEMS AND SOLUTIONS

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University Dropout Problems and Solutions

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Abstract

High dropout rates are often thought to signal inefficient education, but students can drop out of optional higher education only if they previously chose to enroll. This paper shows that dropout is more likely when higher uncertainty increases the probability of news that offset an expectation at enrollment that completion would be better than drop out. Higher uncertainty also increases the value of the option to drop out, so opportunities to enroll and possibly drop out are more valuable ex ante and average educational outcomes are better ex post for degree programs and groups of students with higher uncertainty and efficiently more frequent dropout. Poor information at enrolment, liquidity constraints, and other imperfections can also explain high dropout rates, but attempts to remedy dropout symptoms rather than their underlying causes can introduce new inefficiencies.

JEL: I22, I28

Keywords: Option value, Higher education dropout, Limited information, Rational choice.

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1 Introduction

Policy reports and statistical research abundantly document that dropout varies across degree programs and student populations.[1] Public opinion and policy-makers view high dropout rates, such as those observed in American and Italian public universities, as an indicator of poor educational performance. However, students can drop out of non-mandatory degree programs only if they previously chose to enroll. Understanding why they did enroll is key to interpreting dropout, which can be the optimal choice, in light of new information, after an enrollment choice that was a worthwhile experiment.[2]

This paper brings two insights to bear on how different problems and solutions determine enrollment and dropout across educational opportunities and groups of students. One is that dropout is optional, and options are well known to be more valuable when they make it possible to exploit better good news and escape worse bad news. The other is that a wider spread of possible news makes it more likely that they will be large enough to reverse an expectation at enrollment that completion would be better than dropout.

Viewing enrollment and dropout as optional choices triggered by early and updated expectations of educational costs and benefits offers a new perspective on different dropout intensities across degree programs and student populations. If students who drop out more frequently (such as disadvantaged ones at public universities) are *ex ante* less sure about the costs and benefits of higher education, the dropout choices triggered by their larger revisions of expectations are efficient, and opportunities for them to enroll and possibly drop out are more valuable. Liquidity constraints and imperfect information can imply inefficiently high dropout rates, but attempts to remedy dropout symptoms rather than their underlying causes can introduce new inefficiencies.

The formal derivations suppose that the distribution as of enrollment time of future educational outcomes in utility terms is continuous and well approximated by a stable function of their mean and variance. This approach, described and illustrated in Appendix, allows Section 2 to

[1] See e.g. Vossensteyn et al. (2015) for European countries, Bound et al. (2010) for the US, Ghignoni (2016) and Contini, Cugnata, and Scagni (2018) for Italy.

[2] As pointed out among others by Comay, Melnik, and Pollatschek (1976), Manski (1989), Ozdagli and Trachter (2014), as well as by Alfred Lord Tennyson ("It's better to have tried and failed than to live life wondering what would've happened if I had tried") and, somewhat more cryptically, Bob Dylan ("She knows there's no success like failure, and that failure's no success at all").

show concisely that wider dispersion of possible educational outcomes makes dropout more likely *ex post* if at enrollment it was expected to be worse than completion, and makes enrollment more appealing *ex ante*. Aggregating individual choices, Section 3 relates expectations and uncertainty to enrollment, dropout, and completion in an educational opportunity and its potential student population. Enrollment is higher and dropout less frequent in degree programs and population segments with better expected completion outcomes. An incomplete education can be valuable, however, and better expected dropout outcomes increase both enrollment and dropout. More interestingly, both enrollment and dropout also increase, and expected and average educational outcomes are better, when uncertainty is higher around enrollment-time expectations that are worse for dropout than for completion. Section 4 concludes discussing implications for empirical research and for corrective policies.

2 Individual choices

It is convenient to introduce notation for the discounted expectation, by an individual who is considering enrollment in a degree program, of educational and labor market utility costs and benefits optimized with respect to any choice other than those to enroll and drop out. It will be denoted y_d if the individual enrolls and drops out at some later time, y_c if the individual completes the degree program (or continues rather than drop out), and y_o if the individual does not enroll. Appendix A spells out the formal definition of these discounted expectations, dubbed "payoffs" for brevity below, and Appendix B inspects simple worked-out examples.

This notation considerably streamlines the derivations that in what follows find some key implications of educational and labor market shocks to depend on

$$y_d < y_c : \tag{1}$$

expectations based on information available at enrollment, without taking into account the option to dropout, are worse for dropout than for continuation or completion. This plausible condition supports the popular notion that dropout is wasteful and should be avoided.[3] The

[3] As discussed in some of the following footnotes the model is applicable to situations where this condition is violated, as it may be for individuals who at enrollment expect that dropout, after learning

extent to which y_d falls short of y_c depends on features of educational programs. Those that initially focus on generally useful education offer better dropout payoffs than those that immediately teach and test highly specialized knowledge and skills, and the optimized continuation or completion payoff is better when it is easier to switch major fields within a degree program.

2.1 Dropout

Welfare flow realizations are the same in y_c and y_d before dropout, so they cancel out in their difference. When the student considers dropping out, new information makes the future outlook differ from what was expected at enrollment. Expectations at that time of the value of dropout and completion are additively updated by random variables that have zero expectation as of enrollment time but may be positive or negative in realization. In what follows they are denoted ϵ_d and ϵ_c and dubbed "news." Their standard deviations, or "spreads" for brevity, are denoted σ_d and σ_c.

Bad news about continuation and/or good news about dropout can upturn the *ex ante* expectation (1) that dropout would be worse. When the student considers dropping out it is optimal to do so if $y_c + \epsilon_c < y_d + \epsilon_d$, which as of enrollment time has probability

$$P_d = \text{prob}\left(y_c + \epsilon_c < y_d + \epsilon_d\right) = \text{prob}\left(\epsilon_c - \epsilon_d < y_d - y_c\right) = F\left(\frac{y_d - y_c}{\sigma}\right) \tag{2}$$

where $F(\cdot)$ is the cumulative distribution function of $\epsilon_c - \epsilon_d$ standardized to unit variance by $\sigma = \sqrt{\text{var}(\epsilon_c - \epsilon_d)}$.

As y_d, y_c and σ vary, if $F(\cdot)$ has positive first derivative at the dropout boundary then differentiation of (2) yields

$$\frac{\partial P_d}{\partial y_d} = F'\left(\frac{y_d - y_c}{\sigma}\right)\frac{1}{\sigma} > 0,\ \frac{\partial P_d}{\partial y_c} = -F'\left(\frac{y_d - y_c}{\sigma}\right)\frac{1}{\sigma} < 0,$$
$$\frac{\partial P_d}{\partial \sigma_c} = F'\left(\frac{y_d - y_c}{\sigma}\right)\frac{y_c - y_d}{\sigma^2} > 0 \text{ if } y_d < y_c \tag{3}$$

and establishes

Result 1 ***Intensity of dropout.*** *The probability of dropout is increasing in its payoff y_d;*

some skills or perhaps obtaining an intermediate degree, will be better than completing a degree program.

decreasing in the completion payoff y_c; and, when condition (1) holds, increasing in the spread of possible news after enrollment.

Intuitively, dropout is more likely because news that are negative enough to trigger it have higher probability when their distribution is more spread out.[4]

The variance and higher moments of the mean-zero random variable $\epsilon_c - \epsilon_d$ may differ across degrees and individuals, and the payoffs also depend on characteristics of the educational program and on individual circumstances. Structural features (such as those discussed in Appendix B) generally influence both expectations of educational outcomes and the spread of possible news about them. It is however conceptually possible and useful to distinguish their effects through each channel, and some are plausibly more relevant in one or the other respect. For example, the higher individual discount rate implied by credit rationing decreases the expected value y_c of continuation and completion, which entail additional immediate costs and only later provide benefits, and makes dropout more likely. If the costs and benefits of higher education happen to be more uncertain for more stringently constrained individuals, their dropout probability is unambiguously higher.

2.2 Enrollment

Before blaming high uncertainty as a determinant of undesirable dropout, recall that individuals who drop out previously chose to enroll, and consider the role of uncertainty in determining that choice. A student enrolls if

$$\begin{aligned} V(\cdot) &= \left(y_c + E\left[\epsilon_c | \epsilon_d - \epsilon_c < y_c - y_d\right]\right)(1 - P_d) + \left(y_d + E\left[\epsilon_d | \epsilon_c - \epsilon_d < y_d - y_c\right]\right) P_d \\ &= y_c + E\left[\epsilon_c | \epsilon_c - \epsilon_d > y_d - y_c\right](1 - P_d) + E\left[\epsilon_d | \epsilon_c - \epsilon_d < y_d - y_c\right] P_d + (y_d - y_c) P_d \end{aligned} \tag{4}$$

exceeds the payoff y_o of non-enrollment. Under certainty, $\epsilon_c = \epsilon_d \equiv 0$, and condition (1) makes it optimal to enroll and complete when $y_c > y_o$.[5] Under uncertainty, by condition (1) the student

[4] If (1) does not hold the dropout probability is 100% at $\sigma = 0$ and declines in σ- Students who do not expect completion (or an advanced degree) to be better for them than an incomplete (or basic) degree are more likely to complete (or continue) when enrollment provides more information.

[5] If condition (1) does not hold, when $\max\{y_c, y_d\} = y_d > y_o$ it is optimal to enroll and subsequently drop out with certainty.

expects that it will be better to complete than to drop out, but is aware that news worse for continuation than for dropout may make dropout the better choice. Hence, mean-zero news can be so spread out as to make enrollment optimal when $y_c < y_o$.

To characterize formally how the enrollment value depends on the problem's parameters it will be useful to recall that

$$\frac{d}{dk}\left(\int_k^\infty z\, dP\left(z\right) + kP\left(k\right)\right) = -kP'\left(k\right) + P\left(k\right) + kP'\left(k\right) = P(k) \tag{5}$$

if $P(\cdot)$ is differentiable at k. In what follows applications of this mathematical fact have a straightforward economic interpretation: the problem's parameters do not to first order affect its value through changes of the optimal dropout choice.

Consider first the special case where only completion is uncertain, so that

$$V(\cdot) = \left(y_c + E\left[\epsilon_c|\epsilon_c > y_d - y_c\right]\right)(1 - P_d) + y_d P_d.$$

An enrolled individual drops out and obtains y_d with probability P_d, completes and obtains $y_c + E\left[\epsilon_c|\epsilon_c > y_d - y_c\right] > y_d$ with probability $(1 - P_d)$. Hence, $y_o < y_d$ is a sufficient condition for enrollment to be optimal. If y_c and y_d are both less than y_o enrollment would entail an expected loss if it were irreversible, but the option to observe ϵ_c and possibly drop out adds to enrollment a value that is non-negative and, if $y_c < y_o$, must be positive for the enrolled.

The value of enrollment includes the excess over y_c of the expected outcome conditional on not dropping out, $E\left[\epsilon_c|\epsilon_c > y_d - y_c\right](1 - P_d) = \int_{y_d - y_c}^\infty \epsilon_c dF\left(\epsilon_c/\sigma_c\right)$. Because $\epsilon_d \equiv 0$, $F(\cdot)$ is the distribution of ϵ_c/σ_c and $P_d = F\left(\left(y_d - y_c\right)/\sigma_c\right)$, so

$$V(\cdot) = y_c + \int_{y_d - y_c}^\infty \epsilon_c\, dF\left(\frac{\epsilon_c}{\sigma_c}\right) + \left(y_d - y_c\right) F\left(\frac{y_d - y_c}{\sigma_c}\right). \tag{6}$$

To see that random news truncated by optimal dropout are larger in expectation when more information is revealed between enrollment and dropout, change variables to $z = \epsilon_c/\sigma_c$, $\epsilon_c = \sigma_c z$ to obtain

$$V(\cdot) = y_c + \sigma_c \int\limits_{\frac{y_d - y_c}{\sigma_c}}^{\infty} zF'(z)dz + \left(y_d - y_c\right) F\left(\frac{y_d - y_c}{\sigma_c}\right).$$

By (5) with $k = (y_d - y_c)/\sigma_c$, $P(z) = F(z)$, and using that $dk/d\sigma_c = -(y_d - y_c)/\sigma_c^2$,

$$\frac{d}{d\sigma_c}\left(\int\limits_{\frac{y_d-y_c}{\sigma_c}}^{\infty} zF'(z)dz + \frac{y_d-y_c}{\sigma_c}F\left(\frac{y_d-y_c}{\sigma_c}\right)\right) = -\frac{y_d-y_c}{\sigma_c^2}F\left(\frac{y_d-y_c}{\sigma_c}\right),$$

so

$$\begin{aligned}\frac{\partial V(\cdot)}{\partial \sigma_c} &= \int\limits_{\frac{y_d-y_c}{\sigma_c}}^{\infty} zF'(z)dz + \frac{y_d-y_c}{\sigma_c}F\left(\frac{y_d-y_c}{\sigma_c}\right) + \sigma_c F\left(\frac{y_d-y_c}{\sigma_c}\right)\left(-\frac{y_d-y_c}{\sigma_c^2}\right) \\ &= \int\limits_{\frac{y_d-y_c}{\sigma_c}}^{\infty} zF'(z)dz \geq 0. \qquad (7)\end{aligned}$$

The inequality follows from $\int_{-\infty}^{\infty} zF'(z)dz = 0$ because news have mean zero, and is strict if $\int_{-\infty}^{y_d-y_c} F'(\epsilon_c/\sigma_c)d\epsilon_c > 0$, i.e. dropout has positive probability. Uncertainty makes dropout more likely, but when the dropout choice is optimal this has no first-order implications for the value of enrollment, which varies by the positive expectation of truncated completion news.

To see the effects of payoff expectations, note that

$$\frac{d}{d(y_d - y_c)}\left(\int_{y_d-y_c}^{\infty} \epsilon_c \, dF\left(\frac{\epsilon_c}{\sigma_c}\right) + (y_d - y_c)F\left(\frac{y_d-y_c}{\sigma_c}\right)\right) = F\left(\frac{y_d-y_c}{\sigma_c}\right) = P_d$$

by (5) with $k = y_d - y_c$, $P(z) = F(z/\sigma_c)$, and differentiate (6) using $d(y_d - y_c)/dy_c = -1$, $d(y_d - y_c)/dy_d = 1$ to obtain

$$\frac{\partial V(\cdot)}{\partial y_d} = P_d, \ \frac{\partial V(\cdot)}{\partial y_c} = 1 - P_d. \qquad (8)$$

Optimality of the dropout choice again implies that different cutoffs have no first-order effects for the optimized enrollment value, as the variation of the truncated expected completion outcome offsets the changing probability of obtaining only the dropout payoff.

If instead only the dropout outcome is random and $G(\cdot)$ is the standardized distribution of ϵ_d, then $P_d =$prob$(\epsilon_d > y_c - y_d) = (1 - G((y_c - y_d)/\sigma_d))$, and

$$\begin{aligned}V(\cdot) &= y_c + E[\epsilon_d | \epsilon_d > y_c - y_d]\left(1 - G\left(\frac{y_c-y_d}{\sigma_d}\right)\right) + (y_d - y_c)\left(1 - G\left(\frac{y_c-y_d}{\sigma_d}\right)\right) \\ &= y_d + \int_{y_c-y_d}^{\infty} \epsilon_d \, dG\left(\frac{\epsilon_d}{\sigma_d}\right) + (y_c - y_d)G\left(\frac{y_c-y_d}{\sigma_d}\right). \qquad (9)\end{aligned}$$

Steps similar to those leading to (7) show that $\partial V(\cdot)/\partial\sigma_d$ is positive when the probability of dropout is positive, and using (5) with $k = y_d - y_c$ and $P(z) = G(z/\sigma_d)$ differentiation of (9) with $d(y_c - y_d)/dy_c = 1$, $d(y_c - y_d)/dy_d = -1$ again yields (8).

Consider next the general case where both ϵ_c and ϵ_d are random and, for notational simplicity, independent. If $H(\epsilon_c/\sigma_c)$ is the distribution of ϵ_c, the expected value conditional on the realization of ϵ_d is $y_c + \int_{\epsilon_d+y_d-y_c}^{\infty} \epsilon_c \, dH(\epsilon_c/\sigma_c) + (\epsilon_d + y_d - y_c) H((\epsilon_d + y_d - y_c)/\sigma_c)$. Taking expectations over ϵ_d,

$$V(\cdot) = \int_{\epsilon_d=-\infty}^{\infty} \left(y_c + \int_{\epsilon_d+y_d-y_c}^{\infty} \epsilon_c \, dH\left(\frac{\epsilon_c}{\sigma_c}\right) + (\epsilon_d + y_d - y_c) H\left(\frac{\epsilon_d + y_d - y_c}{\sigma_c}\right)\right) dG\left(\frac{\epsilon_d}{\sigma_d}\right) \quad (10)$$

$$= \int_{\epsilon_c=-\infty}^{\infty} \left(y_c + \int_{y_c-y_d}^{\infty} \epsilon_d \, dG\left(\frac{\epsilon_d}{\sigma_d}\right) + (y_d - y_c - \epsilon_c) G\left(\frac{y_d - y_c - \epsilon_c}{\sigma_d}\right)\right) dH\left(\frac{\epsilon_c}{\sigma_c}\right), \quad (11)$$

where the second equality follows from fixing the realization of ϵ_c and taking expectations across its distribution. Derivations like those leading to (7) show that in each of these expressions the inner integral increases in the relevant spread at each realization of the variable in the outer integral, and so does its expectation across those realizations. Hence, a larger spread of news about either dropout or completion outcomes increases the value of enrollment, establishing

Result 2 ***Uncertainty and the value of enrollment.*** *When the probability of dropout is larger than zero and less than unity, a wider spread of news increases the value of enrollment.*

By (8), the derivatives with respect to $\epsilon_d + y_d$ and y_c of the inner integral in (10) are the dropout and completion probabilities conditional on ϵ_d. Their expectations over ϵ_d are the overall probabilities of dropout and completion, hence

Result 3 ***Conditional expectations and the value of enrollment.*** *When the probability of dropout is larger than zero and less than unity, the value of enrollment increases in the dropout payoff and in the completion payoff.*

Like uncertainty, the dropout and completion payoffs depend on characteristics of educational programs, such as their length and labor-market value, and on individual circumstances. For example, if the balance of education's expected costs and benefits is initially negative and

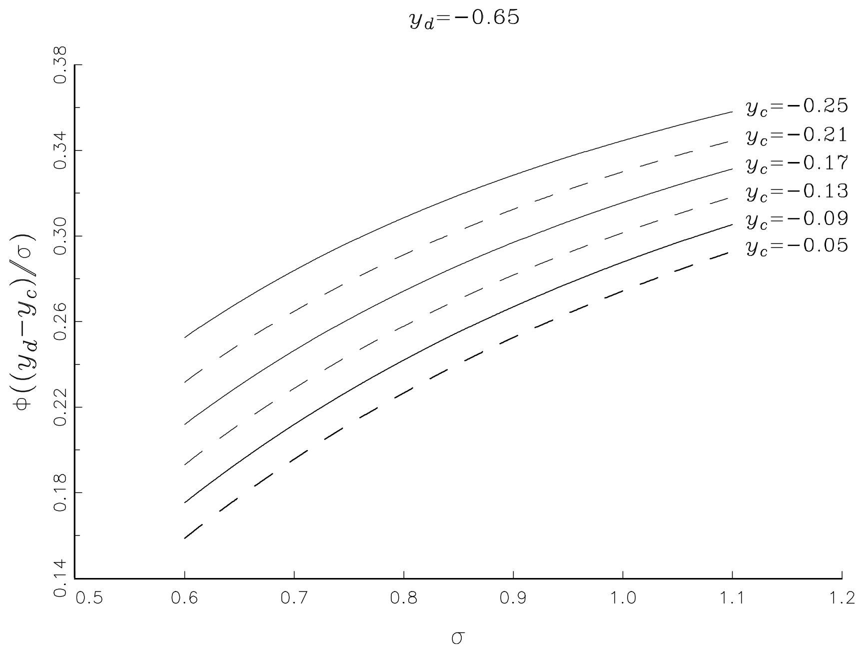

Figure 1: Dropout probability.

becomes positive in the more or less distant future, then binding liquidity constraints decrease both y_c and y_d relative to y_o, and discourage enrollment. But if the costs and benefits of optional education are more random for more stringently constrained individuals, their larger σ make enrollment more attractive.

2.3 Normally distributed news

The derivations above express the probability of dropout as a function of the payoff difference expected at enrollment, standardized by the spread of the news that update those expectations, and rely on that function remaining the same as payoffs and spreads vary across educational programs and groups of students. This cannot be exactly true in general, because variation of the problem's structural parameters typically changes the functional form of the news distribution (see Appendix B for some tractable examples). While partial derivatives may be larger or smaller in absolute size in different structural specifications, their sign is that established by the derivations above as long as the structural problem's realized educational values have a probability distribution that has finite variance and positive density at the dropout trigger.

A complicated reality would not necessarily be better represented by specific forms of utility and other functions, and it is a useful approximation to suppose that as structural features of the problem vary the density of standardized news remains the same, and is symmetric around its zero mean.[6] If welfare realizations are driven by many shocks over multiple periods and along numerous dimensions, normality of that distribution is plausible, and as convenient here as in finance and econometrics. The standard Gaussian density

$$F'(z) = \frac{1}{\sqrt{2\pi}} \exp(-z^2/2) \equiv \varphi(z)$$

is always positive, so dropout is possible for all the enrolled. Figure 1 illustrates Result 1 under normality. Dropout is less likely when the completion outcome y_c is larger, more likely when it is more uncertain and (1) holds. Under normality the dropout probability is about 30% when $(y_d - y_c)/\sigma = -0.55$. i.e. the completion payoff exceeds the dropout payoff by around half a standard deviation of the possible news. The derivations in Appendix B suggest that these parameters approximate a problem where the dropout payoff is 65 log points below a non-enrollment payoff normalized to $y_o = 0$, and the completion payoff falls somewhat short of y_o.

The difference of news about dropout and completion outcomes is normally distributed when both are normally distributed, and Appendix C shows that the value of enrollment is

$$V(y_d, y_c, \sigma) = y_c + \sigma\varphi\left(\tfrac{y_d - y_c}{\sigma}\right) + (y_d - y_c)\,\Phi\left(\tfrac{y_d - y_c}{\sigma}\right) \tag{12}$$

for $\Phi(\cdot)$ the standard Gaussian probability function. The dropout probability is bounded away from zero and unity, so y_d and y_c strictly increase the value of enrollment (Result 3) and so does σ (Result 2): with $\int_y^\infty z \exp(-z^2/2)dz = \exp\left(-y^2/2\right)$ the expectation of the truncated distribution in (7) coincides with the normal density evaluated at the lower limit of integration, hence

$$\frac{\partial V(\cdot)}{\partial \sigma} = \int_{\frac{y_d - y_c}{\sigma_c}}^{\infty} z\varphi(z)dz = \varphi\left(\tfrac{y_d - y_c}{\sigma}\right) \tag{13}$$

is strictly positive and tends to $\varphi(0) = 1/\sqrt{2\pi}$ as $\sigma \to \infty$. Intuitively, $V(\cdot)$ increases without

[6] Symmetry conveniently implies that $F(z) = 1 - F(-z)$ and that dropout has less than 50% probability if condition (1) holds. Non-completion is in the order of 35-50% overall in the US and in Italy (Bound et al., 2010; Ghignoni, 2016), but exceeds 50% for some degrees and groups of students. This may indicate that in some cases the distribution of news is asymmetric or condition (1) is violated.

bound as σ grows because the value of the option to escape bad news and exploit good news becomes arbitrarily large as very favorable or unfavorable information may arrive with positive probability. Enrollment is not optimal at $\sigma = 0$ when dropout and completion payoffs are both lower than that of non-enrollment, but a sufficiently large yet finite σ can bring the value of enrolment above any y_o.

Enrollment is optimal when $V(y_d, y_c, \sigma) > y_o$.[7] Figure 2 displays numerical solutions for the sets of y_d, y_c, σ parameters that equate (12) to a given y_o outside option. In the $\sigma = 0$ certainty case, enrollment is optimal when $y_d > y_o$ for all y_c, and also when $y_c > y_o$ for all y_d. When neither y_c nor y_d exceed y_o, a larger σ adds option value to the enrollment choice and a smaller y_d suffices to make enrollment optimal for each y_c, along lines with increasingly negative slope. The slope becomes vertical as y_c approaches y_o, because if $y_c > y_o$ then the value of enrolment exceeds y_o for all y_d and σ, with certainty when $\epsilon_d \equiv 0$, and always in expectations where symmetrically distributed news around both y_c and y_d cancel out.

The same increasing σ that in Figure 1 implies more intense dropout also increases the value of enrollment in Figure 3, with nearly constant slope because the normal density $\varphi\left(\left(y_d - y_c\right)/\sigma\right)$ varies little in the figure. The slope of the relationship between enrollment value and the spread of news can be different in other parameter ranges, and for distributions other than the normal. By the derivations that lead to (7), it is positive for all the news distributions that have positive density at the dropout cutoff, because a fatter positive tail increases the mean of the truncated news distribution. This option-value effect of individual uncertainty improves average educational outcomes in the following characterization of aggregate enrollment and dropout.

3 Enrollment, dropout, and educational value added

The above characterization of individual choices has straightforward implications for covariation of enrollment and dropout across degree programs and heterogeneous individuals. When news

[7] If individuals are choosing between educational and work opportunities that all allow choices at future times, then y_o can be expressed in the form of (12). Results for enrollment and dropout choices at specific points in time are valid in multiple-alternatives and multiple-period extensions where the preferred choices and timing are those that offer better payoffs and/or option values than available alternatives.

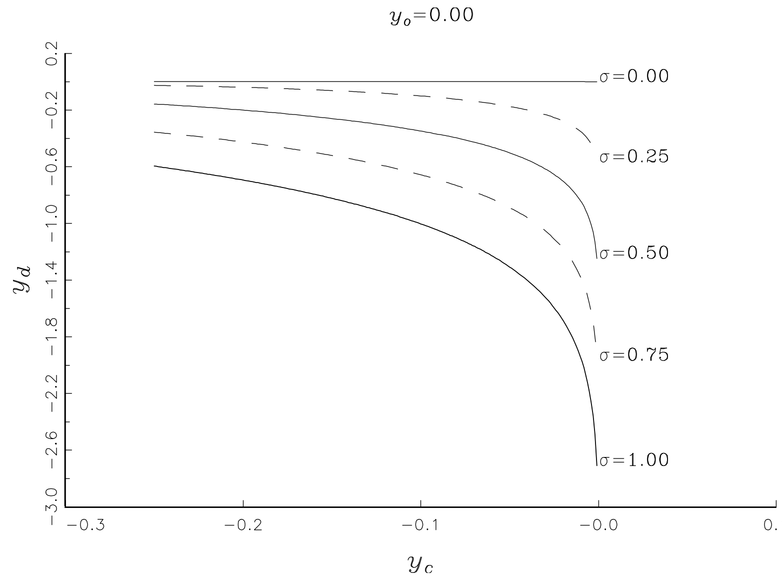

Figure 2: Individual enrollment is optimal for payoff pairs y_c, y_d above and to the right of lines drawn for various degrees of uncertainty σ.

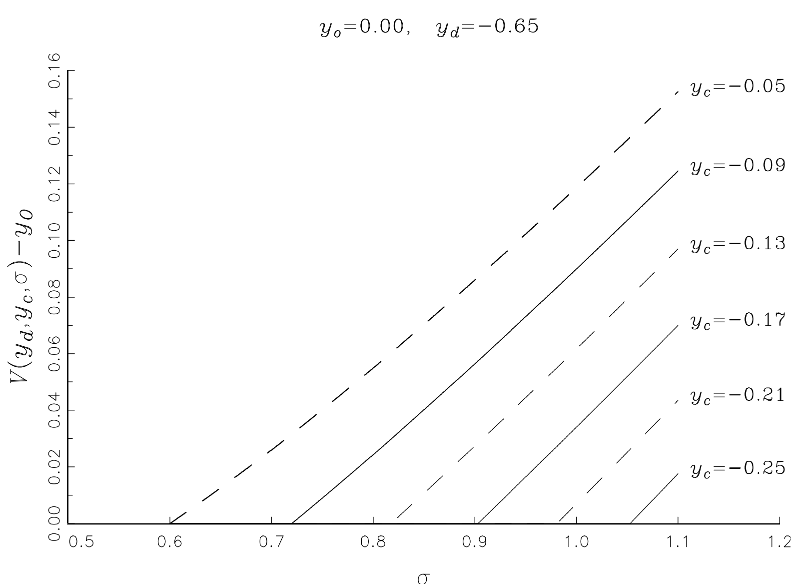

Figure 3: Value added of enrollment opportunity.

are normally distributed, the excess value of enrollment over non-enrollment is expected to be

$$\begin{aligned} V(y_d, y_c, \sigma) - y_o &= \left(\tfrac{y_c - y_o}{\sigma} + \varphi\left(\tfrac{y_d - y_c}{\sigma}\right) + \tfrac{y_d - y_c}{\sigma}\Phi\left(\tfrac{y_d - y_c}{\sigma}\right)\right)\sigma \\ &= \left(\tilde{y}_o + \varphi\left(\tilde{y}_d\right) + \tilde{y}_d \Phi\left(\tilde{y}_d\right)\right)\sigma \qquad (14) \\ &\text{for } \tilde{y}_d \equiv \frac{y_d - y_c}{\sigma},\ \tilde{y}_o \equiv \frac{y_o - y_c}{\sigma}. \end{aligned}$$

This expression depends on payoff differences $y_o - y_c$ scaled by the spread σ. Both generally vary across individuals, who enroll when the expectation and uncertainty parameters introduced in the previous section make (14) positive. This condition, like the dropout probability $\Phi\left(\tilde{y}_d\right)$, depends on scaled payoff differences: the value added of enrollment (14) increases in $\tilde{y}_o$ as well as in $\tilde{y}_d$, because $\varphi'\left(z\right) = -z\exp(-z^2/2)/\sqrt{2\pi} = -z\varphi\left(z\right)$ cancel out in $d[\varphi\left(\tilde{y}_d\right) + \tilde{y}_d\Phi\left(\tilde{y}_d\right)]/d\tilde{y}_d = \Phi\left(\tilde{y}_d\right) > 0$ for the same optimal-dropout reasons that deliver the simple form of (8).

The sign of $\tilde{y}_o$ depends on whether degree completion would be expected to be more or less valuable than not enrolling in the absence of the dropout option. The expression on the right-hand side of (14) also depends on the scaled payoff difference $\tilde{y}_d$, which is negative by (1) but less negative when y_c is smaller or y_d is larger. At given expectations, a larger σ reduces the absolute value of $\tilde{y}_d$ and $\tilde{y}_o$.

Aggregate enrollment and dropout are readily characterized using the Results above, which are valid more generally than in the tractable case of normally distributed news. To pin down enrollment rates at specific educational programs by specific populations, either or both of $\tilde{y}_d$ and $\tilde{y}_o$ must be heterogeneous across individuals. For those who enroll, $\tilde{y}_d$ and $\tilde{y}_o$ imply $\varphi\left(\tilde{y}_d\right) + \tilde{y}_d\Phi\left(\tilde{y}_d\right) > \tilde{y}_o$, so the population enrollment fraction is the integral over the $\tilde{y}_d$ marginal density of the $\tilde{y}_o|\tilde{y}_d$ conditional cumulative distribution. That calculation is feasible for any continuous cross-sectional distribution of the $\tilde{y}_d$ and $\tilde{y}_o$ scaled differences, and is again relatively tractable when that distribution is normal (see Appendix D for derivations). The top panel of Figure 4 plots the $\tilde{y}_o = \varphi\left(\tilde{y}_d\right) + \tilde{y}_d\Phi\left(\tilde{y}_d\right)$ line and bivariate normal density contours. Its bottom panel plots the dropout rate $\Phi\left(\tilde{y}_d\right)$, the enrollment trigger, and the resulting completion rate along with the marginal density of $\tilde{y}_d$. The figure also reports the average educational value added, computed by numerical integration of the positive values of the function (14) of $\tilde{y}_d$ and $\tilde{y}_o$, weighted by the bivariate normal density, hence averaging realized outcomes of both

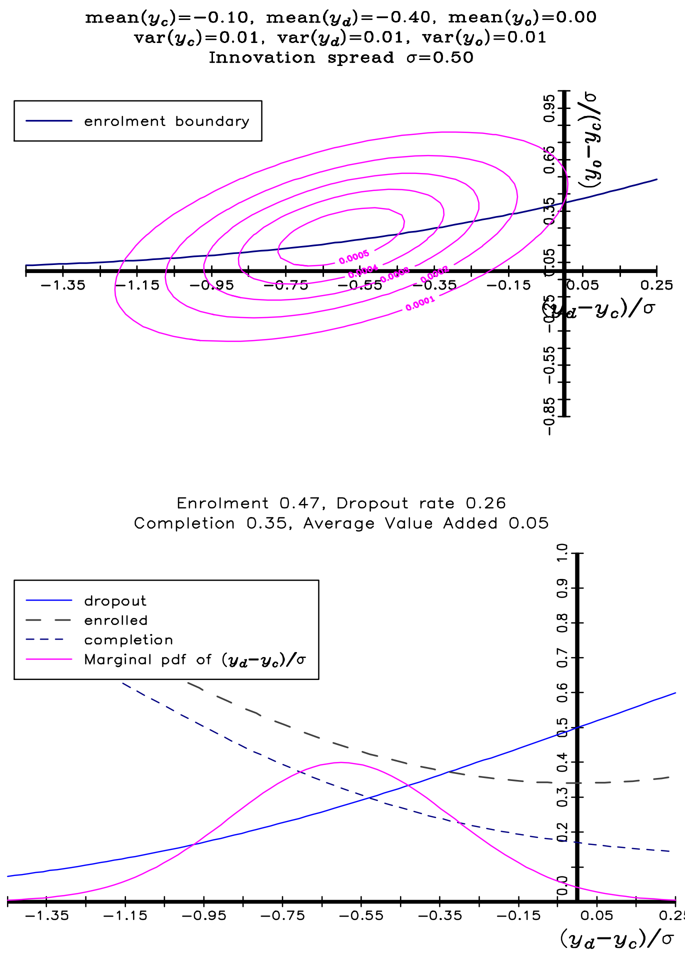

Figure 4: Distribution of bivariate normal scaled expected outcome differences, and conditional and unconditional enrollment and dropout rates.

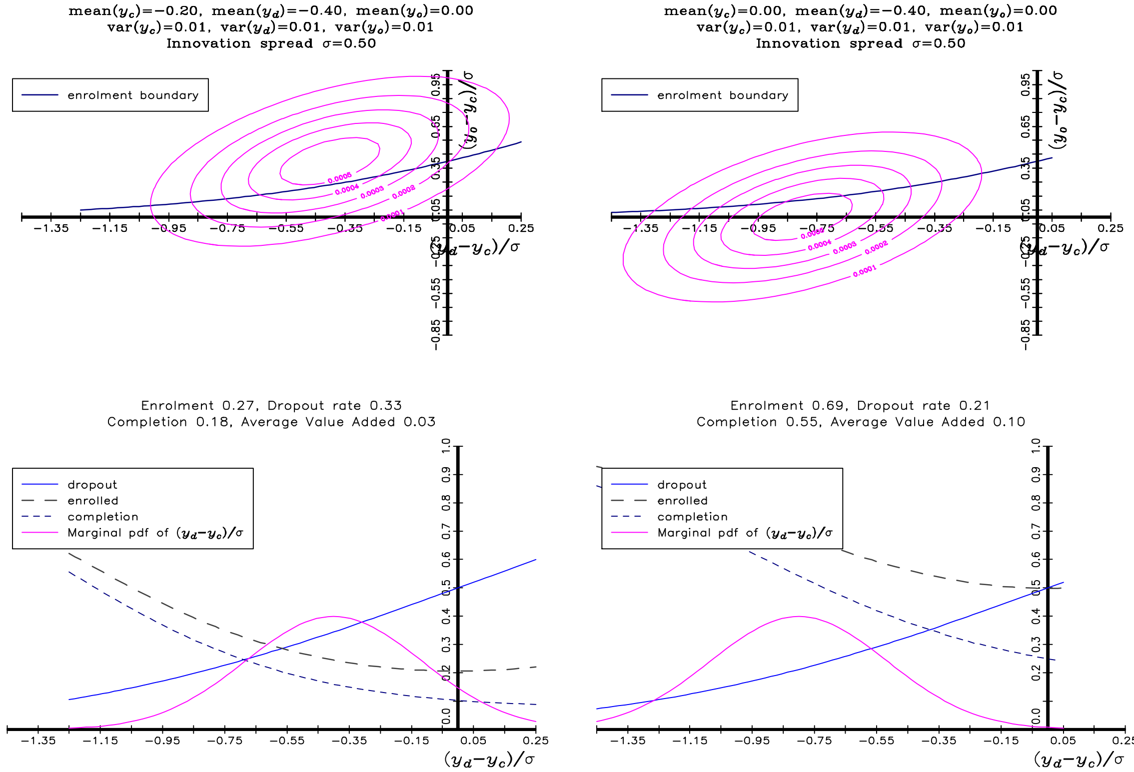

Figure 5: Implications of average completion payoff for enrollment, dropout, completion rates and average value added.

completers and dropouts among the enrolled.

Figure 5 illustrates the effects of the average completion payoff, which is more positive on the right than on the left. In the top panel the distribution of scaled payoff differences shifts diagonally down and to the left, so a larger fraction of the source population enrolls. In the bottom panel, the dropout probability is increasing, so the total dropout rate declines. The effects of y_d, shown in Figure 6, are similarly intuitive. A more positive dropout payoff shifts the bivariate distribution horizontally to the right. Because the enrollment cutoff is increasing, a larger fraction of the source population enrolls. Because in the bottom panel the dropout probability is increasing, the total dropout rate increases. In both figures, the expected and average value added (14) is larger when the distribution of its determinants has more positive means.

The same reasoning is valid for more general cross-section distributions than the bivariate normal used to illustrate these effects, which follow from the equally intuitive Result 3, and establishes

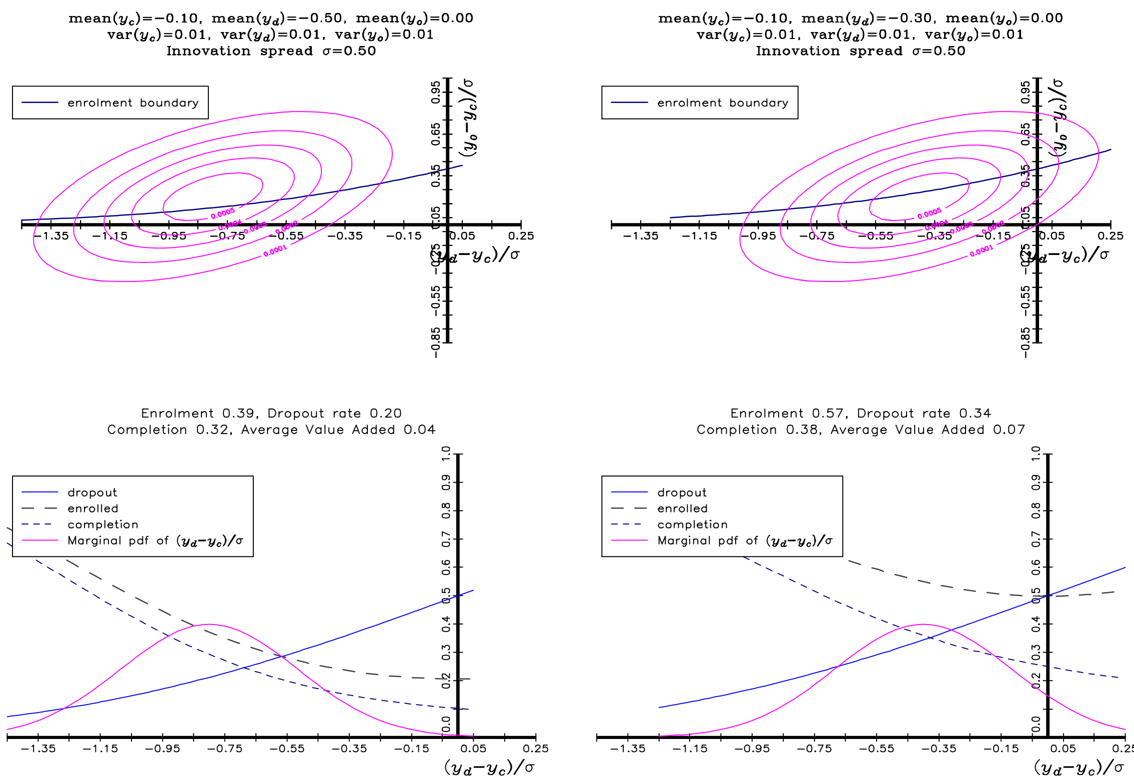

Figure 6: Implications of average dropout payoff for enrollment, dropout, completion rates and average value added.

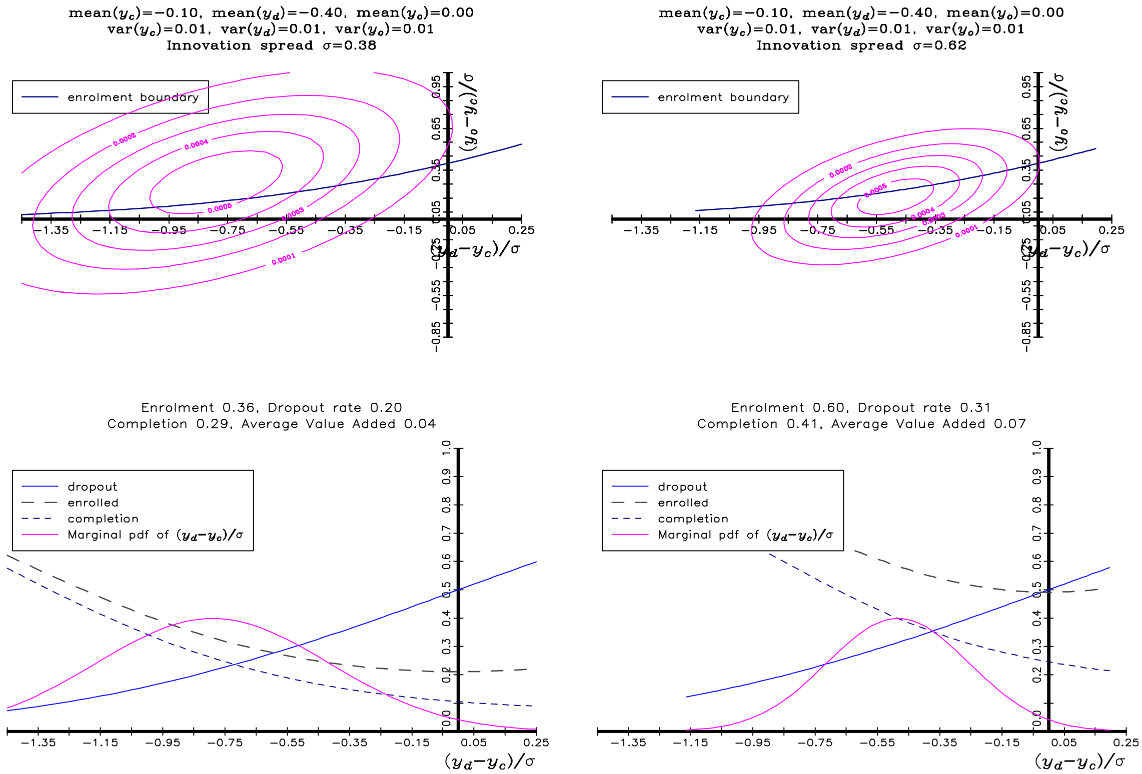

Figure 7: Implications of uncertainty around payoffs for enrollment, dropout, completion, and average value added.

Result 4 ***Conditional expectations and aggregate outcomes.*** *Across student populations and educational opportunities, a larger average completion payoff reduces dropout, a larger average dropout payoff increases dropout, and both increase enrollment and the average value added of education.*

The non-enrollment value y_o is set to zero in the figures, but its implications are a simple composition of those of y_d and y_c: an increase of y_o has the same effect as a decline of both y_d and y_c, and reduces $\tilde{y}_o$ leaving $\tilde{y}_d$ unchanged. The bivariate distribution shifts vertically upwards, and enrollment and average value added both decline. The population marginal distribution of $\tilde{y}_d$ remains unchanged but dropout declines through a composition effect, because $\tilde{y}_d$ is more negative among the enrolled.

The implications of uncertainty around individual expectations are less obvious. Moving from the left to the right in Figure 7 a larger σ shrinks the distribution of scaled payoff differences and, as (1) holds for almost all the population, moves it to the right.[8] The enrollment cutoff is increasing in $\tilde{y}_d$, so a larger fraction of the source population enrolls. In the bottom panel the dropout probability is increasing in $\tilde{y}_d$, so the dropout rate increases.[9] More uncertainty encourages enrollment by Result 2 and improves average completed payoffs even as, by Result 1, it increases dropout among enrolled students.

A larger σ increases overall completion for the parameters used in plotting Figure 7 but need not do so in general, because it increases dropout as well as enrollment. The strength of its enrollment effect depends on the cross-sectional distribution of payoffs: for the normal and other plausible distributions it declines as enrollment increases, and can be dominated by the dropout effect of σ, which by inspection of (13) is strongest at $\tilde{y}_d = 0$ where the normal density is maximal.

The numerical average of the value-added expression (14) also increases with σ in Figure 7, and it is not difficult to see that this is a general implication. Value added is zero for the non-enrolled, positive for those who enroll: a larger σ makes it positive for individuals who would not have enrolled, and increases it when it is already positive at the initial σ. As idiosyncratic *ex post* shocks average out across the population, this establishes

Result 5 ***Uncertainty and aggregate value added.*** *When dropout is possible, the aggregate value added of an educational opportunity is higher if enrollment resolves more pronounced uncertainty about individual educational outcomes.*

A larger σ increases enrollment in the population until, for the marginal individual who rationally chooses to enroll and possibly drop out, the larger expected completion outcome implied by the option to take advantage of good news and escape bad news exactly offsets expected welfare losses in the event of dropout. As it also improves expected and average realized outcomes for inframarginal enrolled individuals, average educational outcomes are better when stronger uncertainty generates more valuable dropout options and, if (1) holds, higher dropout rates.

[8] Condition (1) does not hold in the region to the right of the vertical axis in the figure, where the normal density of payoffs is positive in the normal case: some individuals do expect dropout to be better than completion.

[9] Where condition (1) fails a larger σ moves the distribution of $\tilde{y}_d$ the the left, and reduces dropout.

It may sound puzzling, even to readers of academic journals that treat high rejection rates as a badge of honor, that stronger uncertainty around expected *ex post* educational outcomes attracts more enrollment and, despite higher dropout, implies a larger aggregate value added of educational opportunities. The result should not be misconstrued to mean that riskiness is an attractive feature of educational opportunities. Stronger uncertainty about educational outcomes increases not only the average but also the dispersion of labor income and other welfare-relevant variables. Like stock options, dropout options should be valued on a risk-adjusted basis. In the derivations above the relevant spread is that of news about future values at given utility-terms expectations (see Appendix B.1 for illustrative examples). It would also be wrong to think that the results imply that imprecise exams are beneficial. If students do not know their own ability and future opportunities, exams should reveal that information. Adding noise to grades or randomly failing some students would increase dropout, but reduce value added and enrollment.

In this section's derivations and illustrations individual payoffs are statistically and functionally independent of each other, and aggregation simply sums them. In reality, educational technologies are not linear: if returns are decreasing at a given degree program, then stronger enrollment worsens the marginal educational payoffs y_c, and increases dropout. One way to reduce dropout is to prevent enrollment. In a more selective program, $y_c - y_d$ is more positive on average, and dropout is less likely. Screening is unavoidably imprecise, however, and high-quality institutions competing for high-quality students may find it less costly and more effective to screen after rather than before enrollment.[10] As long as dropout is an unforced student choice made in light of information that arrives after accepting admission and enrolling, the model's perspective and insights remain qualitatively relevant for even the most selective degree programs.[11]

[10] Some spectacularly successful completing students do emerge from high-quality degree programs that, like the University of Chicago's among top Economics Ph.D. in the 1980s, admit students with relatively little selection and obtain relatively low completion rates.

[11] About 2 per cent of Harvard University undergraduates fail to complete in 6 years. The relative appeal of dropout had very positive realizations for Bill Gates, Mark Zuckerberg, Matt Damon, Bonnie Raitt, and many less successful individuals. Comprehensive data on the fate of dropouts are rare, but Paul's (2015) analysis of a German training program finds that dropout is an opportunity rather than a problem for many individuals.

4 Dropout: bug or feature?

This paper's perspective on dropout problems and their possible solutions focuses on the means and variances of education's future value from the perspective of students who consider enrollment and rationally know that dropout may later turn out to be optimal.

If students who choose to enroll expect completion to be better than dropout, a wider spread of possible news around those expectations makes it more likely that they will drop out. But while the probability of news bad enough to trigger dropout is higher, so are the probability of good news, the option value of dropout and continuation choices, and the overall value of enrollment. Attempting is a necessary but not sufficient condition for succeeding, and intense dropout may indicate that enrollment resolves stronger uncertainty and efficiently elicits more valuable information.

This perspective suggests that it is interesting to assess empirically whether uncertainty is indeed the main driver of observed dropout, and that concern about high dropout rates is justified only when they can be linked to structural inefficiencies.

4.1 Empirical aspects

Uncertainty-driven dropout variation might be detected inspecting its correlation with previous enrollment across segments of student populations, or in different degree programs, or over time. A negative association of enrollment and subsequent dropout suggests that they are mainly driven by different completion payoffs, and high dropout rates signal defective educational programs. A positive association suggests that they are driven by different dropout payoffs or by different *ex ante* uncertainty: in this case high dropout rates may be associated with widely dispersed outcomes (such as grades, or first-job wages) among enrolled students, and do not indicate that educational technologies should be improved.

Data limitations make it difficult for empirical research to consider enrollment and dropout choices jointly, and it is also difficult to disentangle individual uncertainty from unobserved heterogeneity in the distribution of educational outcomes. Recent empirical work studies dropout as the solution of structural individual problems. Stange (2012) lets students update their own aptitude and taste assessments while pursuing completion, so dropout is a valuable option.

Stinebrickner and Stinebrickner (2012, 2014) model the theoretical and empirical role of heterogeneous progressively updated degrees of confidence in own ability among enrolled students. Athreya and Eberly (2021) let dropout be triggered by changing individual circumstances as well as by academic performance, Chatterjee and Ionescu (2012) assess the role of uninsured drop-out risk in a similar framework, and Hendricks and Oksana (2018) model dropout choices in a setting where degree completion has heterogenous value and is achieved at individual-specific speeds. The models of Stinebrickner and Stinebrickner (2014) and Hendricks and Leukhina (2018) recognize that within non-enrolled, completers, and dropout groups of individuals the observed distribution of wages is selected not only at enrollment by permanent heterogeneity, but also at dropout times by new information about continuously-distributed future wages. While these papers do not focus on uncertainty as a driver of dropout rates, some of their empirical results can and will be discussed briefly below in the context of the present paper's theoretical framework.

4.2 Dropout as a symptom

Concern about high dropout rates, particularly among disadvantaged students, is justified if it is a symptom of underlying imperfections. For example, liquidity constraints both reduce enrollment and imply more intense dropout among those who enroll. Empirical work finds that they are a significant but quantitatively small determinant of higher-education choices (Cameron and Heckman, 2001), and specific evidence on dropout is similar: Hendricks and Leukhina (2018) find that ability estimated from college transcripts is much more important than financial constraints in determining enrollment and completion, and Stinebrickner and Stinebrickner's (2008) survey evidence does not indicate that self-declared liquidity constraints have any effect on dropout.

Another potentially relevant imperfection is lack of insurance. Uninsured idiosyncratic consumption risk makes riskier alternatives less attractive at given expectations, as illustrated by the examples in Appendix B.1, especially for poorer individuals under decreasing risk aversion (unless they can borrow to finance education and can default, or are covered by a social safety net that makes higher education an essentially one-way bet). The relative riskiness of educational and labor market choices is not generally obvious, and risk increases consumption volatility and decreases *ex ante* welfare on both sides of the enrollment and dropout indifference conditions. The enrollment and completion effects of incomplete insurance are in fact empirically small in the model of Chatterjee and Ionescu (2012).

Imprecise or biased information is a potentially more important and empirically plausible reason why individual enrollment and dropout choices deviate from social optimality. While student groups with higher dropout rates may genuinely face more uncertainty about their own ability as well as future financial shocks, their choices can be socially inefficient when they are based on subjective probability distributions other than that which generates the relevant news. The same positive association of enrollment and dropout that uncertainty implies for rational choices can be driven by misinformation about a degree program's difficulty or labor-market value. Dropout-relevant information generated by enrollment may to some extent be gathered by outside observers of students' exam-taking speed and grades (Hendricks and Leukhina, 2018), and also by surveying their expectations and information-gathering strategies (Stinebrickner and Stinebrickner 2012). This type of empirical evidence does suggest that student expectations often differ significantly from average outcomes conditional on their observable characteristics.

4.3 Remedies

Those who have met very many students and remember how their younger selves chose to enroll in higher education may doubt that young people are fully capable of assessing and comparing the values of educational opportunities. Paternalistic schooling mandates are appropriate for small children and for parents who need not choose in their children's best interest. To the extent that nobody need know more than young adults themselves about what is good for them, however, they are entitled to experiment and learn about their own ability. If students know more than outside observers about their individual circumstances their educational choices, albeit poorly informed, are as rational as possible. Unfortunately, not only material resources and information, but also awareness of educational opportunities and ability to exploit them are on average scarce among disadvantaged students. And because those with low ability find it difficult to realize that it is low, self-assessments are empirically inflated at the low end of the ability distribution (Dunning and Kruger, 1999). If personal traits are revealed by behavior and results rather than by observable characteristics, however, it is not clear whether and how low-ability individuals should and could be prevented from trying and failing without also preventing high-ability individuals from trying and succeeding.

Treating a symptom does not cure a disease, and can make it worse. Preventing enrollment of disadvantaged students would reduce dropout at the cost of excluding those among them who would achieve the outstanding results represented in this paper by large positive realizations of continuously-distributed news. Offering financial help or pedagogical assistance to students can reduce dropout rates but encourages enrollment of individuals who are relatively likely to drop out, and may well increase the total number of dropouts. Remedial education is appropriate in mandatory educational programs that provide citizens and workers with essential social and cultural skills and generate positive externalities. But exams that gauge individual ability produce valuable information that should not be obscured by lenient grading and generous pass criteria in order to prevent dropout. In degree programs meant to identify and exploit comparative advantage, dropout can be inefficiently low when policy reduces it for poor performers.

Increasing enrollment and reducing dropout of disadvantaged individuals in optional higher education is not the best way to help them. Correcting the underling imperfections is desirable if asymmetric information restrains enrollment and increases dropout by uninsured or liquidity constrained individuals, and the choices of poorly informed individuals, albeit rational from their point of view, can be socially inefficient. Policy can gather and publish information about dropout (possibly conditional on observable characteristics) in non-selective public degrees, and accredit and regulate degree programs to prevent the ample opportunities for false advertising offered by imprecise and asymmetric information in private education. The most relevant information, however, is as difficult to gather for public policy as for insurance and credit markets. Brief aptitude tests are less expensive but cannot be more precise than long sequences of exams, and need not be preferable to allowing individuals to experiment, enroll, and drop out.

Appendix A

When considering enrollment a potential student has ability A and financial resources M, and knows that when considering dropout ability will be

$$A' = \begin{cases} a_e(A, L, \varepsilon_L) & \text{if enrolled,} \\ a_o(A, L, \varepsilon_L) & \text{otherwise} \end{cases} \tag{A.1}$$

where L is effort and ε_L is a random shock, and resources will be

$$M' = \begin{cases} (M - C - T + \varepsilon_M)(R + \varepsilon_R) & \text{if enrolled,} \\ (M - C + w_o L + \varepsilon_M)(R + \varepsilon_R) & \text{otherwise} \end{cases} \tag{A.2}$$

where C is consumption, T is tuition fees and other educational costs, w_o is the wage, R is the expected gross return on savings, and random variables ε_M and ε_R shock resources and returns.

During the period before the possible dropout decision utility is $u_e(C, L)$ if enrolled, $u_o(C, L)$ if not, both increasing in C and decreasing in L. When deciding whether to enroll the individual knows that L and C are to be chosen optimally, that it may be later be optimal to drop out, and that the future ability (A.1) and resources (A.2) increase the future values $v_c(A', M', \varepsilon_c)$ if continuing with completed or continuing degree, $v_d(A', M', \varepsilon_d)$ if dropping out, and $v_o(A', M', \varepsilon_o)$ if continuing to work, with ε_c, ε_d, and ε_o representing labor market shocks or taste and health shocks that the potential student is aware may be realized. The value functions could be written recursively as the next period's optimized utility flow and continuation value, and it would be possible to specify more detailed choice sets (such as consumption of various goods, drug use, financial portfolio composition).

Introducing a borrowing limit $\bar{M}$, optimal choices are well-defined under standard concavity and single-crossing conditions:

$$\begin{aligned} \{c_o^*, L_o^*\} = \arg\max_{c,L} & \, u_o(C, L) + E\left[v_o(A', M', \varepsilon_o)\right] \\ \text{s.t. } & M' = (M - C + w_o L + \varepsilon_M)(R + \varepsilon_R), \\ & M - C + w_o L \geq \bar{M}, \\ & A' = a_o(A, L, \varepsilon_L), \end{aligned}$$

$$
\begin{aligned}
\{c_e^*, L_e^*\} = \arg\max_{c,L} u_e(C, L) + E\left[v_c(A', M', \varepsilon_c)(1 - P_d) + v_d(A', M', \varepsilon_d)P_d\right] \\
\text{for } P_d = \text{Prob}\left(v_c(A', M', \varepsilon_c) \leq v_d(A', M', \varepsilon_d)\right) \\
\text{s.t. } M' = (M - C - T + \varepsilon_M)(R + \varepsilon_R), \\
M - C - T \geq \bar{M}, \\
A' = a_e(A, L, \varepsilon_L).
\end{aligned}
$$

The main body of the paper defines

$$
\begin{aligned}
u_o(c_o^*, L_o^*) + E\left[v_o(a_o(A, L_o^*, \varepsilon_L), (M - C_o^* + w_o L_o^* + \varepsilon_M)(R + \varepsilon_R), \varepsilon_o)\right] &\equiv y_o, \\
u_e(c_e^*, L_e^*) + E\left[v_c(a_e(A, L_e^*, \varepsilon_L), (M - C_e^* - t + \varepsilon_M)(R + \varepsilon_R), \varepsilon_c)\right] &\equiv y_c, \\
u_e(c_e^*, L_e^*) + E\left[v_d(a_e(A, L_e^*, \varepsilon_L), (M - C_e^* - t + \varepsilon_M)(R + \varepsilon_R), \varepsilon_d)\right] &\equiv y_d, \\
v_c(a_e(A, L_e^*, \varepsilon_L), (M - C_e^* - T + \varepsilon_M)(R + \varepsilon_R), \varepsilon_c) & \\
- E\left[v_c(a_e(A, L_e^*, \varepsilon_L), (M - C_e^* - T + \varepsilon_M)(R + \varepsilon_R), \varepsilon_c)\right] &\equiv \epsilon_c, \\
v_d(a_e(A, L_e^*, \varepsilon_L), (M - C_e^* - T + \varepsilon_M)(R + \varepsilon_R), \varepsilon_d)) & \\
- E\left[v_d(a_e(A, L_e^*, \varepsilon_L), (M - C_e^* - T + \varepsilon_M)(R + \varepsilon_R), \varepsilon_d)\right] &\equiv \epsilon_d.
\end{aligned}
\tag{A.3}
$$

Utility before possible dropout is the same in y_c and y_d, so condition (1) in the main text depends on the expectation as of enrollment of the continuation values. The distributions of $\{\varepsilon_A, \varepsilon_M, \varepsilon_R, \varepsilon_c, \varepsilon_d\}$, the form of the utility and accumulation functions, and the optimal pre-dropout choices determine the distribution of ϵ_c and ϵ_d around their expectation at enrollment time, which is zero by definition. The main text summarizes those structural features with the functional form $F(\cdot)$ of the distribution of $\epsilon_c - \epsilon_d$ standardized by $\sigma = \sqrt{\text{var}(\epsilon_c - \epsilon_d)}$, and supposes that the functional form $F(\cdot)$ remains the same as parametric features of the problem vary. Then, partial derivatives relate enrollment and dropout locally to variation of σ, which increases in the variances of $\{\varepsilon_A, \varepsilon_M, \varepsilon_R, \varepsilon_c, \varepsilon_d\}$, and y_o, y_c, y_d, which also depend on the variance of shocks that enter the relevant functions nonlinearly.

Appendix B

What follows inspects the definitions introduced in Appendix A for some flexible and tractable functional forms assumed in applied work, focusing on randomness of the labor income that a student may earn if not enrolling, completing, or dropping out.

B.1 Risk aversion

To isolate the implications of risk aversion suppose there is no effort choice and no access to financial instruments. With $M' = 0$ consumption when studying coincides with initial resources minus educational costs, and when working coincides with labor income w:

$$u_e(C, L) = \frac{(M - T)^{1-\rho} - 1}{1 - \rho},\ u_0\left(w_0, L\right) = v_j\left(w_j, L\right)(1 + \delta) = \frac{(w_j)^{1-\rho} - 1}{1 - \rho} \text{ for } j = c, d, o, \tag{B.1}$$

where δ is a discount rate and relative risk aversion $\rho \geq 0$ indexes the curvature of utility as a function of consumption (or the degree to which private or public insurance partially smooths out the consumption impact of income shocks).

For $\rho = 0$ choices are risk neutral,

$$y_o = E\left[w_o\right] + \frac{1}{1+\delta} E\left[w_o\right],\ y_c = M - T + \frac{1}{1+\delta} E\left[w_c\right],\ y_d = \ln\left(M - T\right) + \frac{1}{1+\delta} E\left[w_d\right],\ \sigma = \text{var}\left(w_c - w_d\right)$$

and if wages are normally distributed the derivations may proceed as in Section 2.3 of the main text (as negative wage realizations are possible when their distribution is normal, it would be sensible to allow for an option not to work, with implications similar to the dropout option of interest in this paper).

For $\rho = 1$ utility is logarithmic, $u\left(w\right) = \ln(w)$. If $w_j = e^{\mu_j + \sigma_j \varepsilon_j}$ for ε_j random variables with zero expectation then $E\left[u\left(w_j\right)\right] = \mu_j$,

$$y_o = \ln \mu_o + \frac{1}{1+\delta}\mu_o,\ y_c = \ln\left(M - T\right) + \frac{1}{1+\delta}\mu_c,\ y_d = \ln\left(M - T\right) + \frac{1}{1+\delta}\mu_d$$

so the main text's condition (1) reads $\mu_d < \mu_c$, and $\sigma = \text{var}(\epsilon_c - \epsilon_d)$. The variance of log wages does not appear in utility expectations but by Jensen's inequality does influence the expectation of wage levels (and, conversely, higher wage variance decreases risk-averse utility at given wage level expectations). For example,

$$E\left[w_j\right] = e^{\mu_j + \frac{1}{2}\sigma_j^2} \text{ if } \text{Prob}(\varepsilon_j < x) = \Phi(x) \tag{B.2}$$

for $\Phi(\cdot)$ the standard normal distribution, and the derivations of Section 2.3 are again directly applicable.

For other values of ρ dropout occurs if $v_d\left(w_d\right) - v_c\left(w_c\right) = \frac{1}{1-\rho}\left(e^{(\mu_d + \sigma_d \varepsilon_d)(1-\rho)} - e^{(\mu_c + \sigma_c \varepsilon_c)(1-\rho)}\right) > 0$. Its probability does not depend on ρ, and is

$$\text{Prob}\left(\sigma_c \varepsilon_c - \sigma_d \varepsilon_d < \mu_d - \mu_c\right) = \Phi\left(\left(\mu_d - \mu_c\right)/\sigma\right). \tag{B.3}$$

if wages are lognormal as in (B.2). The variance of either wage increases the intensity of dropout when log-wage expectations are lower for dropout than for continuation or completion. The main text's partial derivatives with respect to σ, however, are taken at constant utility expectations: this requires log means to vary and compensate the expected utility impact of σ_d^2 and/or σ_c^2 variation,

$$\mu_d = \bar{\mu}_d - \frac{1}{2}(1-\rho)\,\sigma_d^2,\;\; \mu_c = \bar{\mu}_c - \frac{1}{2}(1-\rho)\,\sigma_c^2, \tag{B.4}$$

so that the value expectations

$$E\left[v_d\left(w_d\right)\right] = \frac{e^{(1-\rho)\bar{\mu}_d}-1}{1-\rho},\quad E\left[v_c\left(w_c\right)\right] = \frac{e^{(1-\rho)\bar{\mu}_c}-1}{1-\rho}$$

do not depend on uncertainty. The main text's condition (1) reads $\bar{\mu}_d < \bar{\mu}_c$, which is not the same as $\mu_d < \mu_c$ if $(1-\rho)\,\sigma_d^2 \neq (1-\rho)\,\sigma_c^2$. The distribution of news about utility values in the form (B.1) is lognormal, and if ε_c and ε_d are uncorrelated its variance is

$$\begin{aligned}
\mathrm{var}\left(\epsilon_d - \epsilon_c\right) &= \frac{1}{(1-\rho)^2}\mathrm{var}\left(e^{(\mu_d+\sigma_d\varepsilon_d)(1-\rho)} - e^{(\mu_c+\sigma_c\varepsilon_c)(1-\rho)}\right)\\
&= \frac{1}{(1-\rho)^2}\left(e^{2\mu_d(1-\rho)+(\sigma_d(1-\rho))^2}\left(e^{(\sigma_d(1-\rho))^2}-1\right) + e^{2\mu_c(1-\rho)+(\sigma_c(1-\rho))^2}\left(e^{(\sigma_c(1-\rho))^2}-1\right)\right)\\
&= \frac{1}{(1-\rho)^2}\left(e^{2\bar{\mu}_d(1-\rho)}\left(e^{(\sigma_d(1-\rho))^2}-1\right) + e^{2\bar{\mu}_c(1-\rho)}\left(e^{(\sigma_c(1-\rho))^2}-1\right)\right)
\end{aligned} \tag{B.5}$$

where the last equality uses (B.4) to keep expected values constant as σ_d or σ_d change the variance of utility-terms news. Setting to zero the total differential of (B.5) with respect to log means and variances it is also possible to keep $\mathrm{var}(\epsilon_d - \epsilon_c)$ locally constant as value expectations have the effects shown in the main text by partial derivatives.

B.2 Effort choice

Still ruling out borrowing and lending, let labor income depend on ability at given wages, and suppose that utility and continuation values feature linear effort supply and unitary relative risk aversion. With

$$u_o\left(C,L\right) = \ln(w_o AL - AL^2),\;\; u_e\left(C,L\right) = \ln(M-T-AL^2),\;\; v_j\left(w_j,L\right)(1+\delta) = 1\ln(w_j A' L - L^2) \text{ for } j=c,d$$

the optimized values of working are

$$\begin{aligned}
v_j\left(w_j,L_j\right) &= \frac{1}{1+\delta}\max_{L_j}\left(wA'L_j - L_j^2\right)\\
&= \frac{1}{1+\delta}\ln\left(\frac{1}{4}\left(A'w_j\right)^2\right),\;\; j=c,d.
\end{aligned} \tag{B.6}$$

Let study effort multiplicatively and randomly influence future ability,

$$A' = \begin{cases} a_e(A,L,\varepsilon_A) = ALe^{\mu_A+\sigma_A\varepsilon_A} & \text{if enrolled,}\\ a_o(A,L,\varepsilon_L) = A & \text{otherwise,}\end{cases} \tag{B.7}$$

and denote $w_j = e^{\mu_j + \sigma_j \varepsilon_j}$, where the log means μ_c, μ_d account for different average wages or a different impact of education in the completer and dropout markets. Inserting (B.7) in (B.6),

$$v_j\left(w_j, L_j\right) = \frac{1}{1+\delta}\left(-\ln(4) + 2\left(\ln\left(L_e\right) + \ln(A/4) + \mu_A + \sigma_A\varepsilon_A + \mu_j + \sigma_j\varepsilon_j\right)\right), \; j = c, d$$

and the dropout probability is given by (B.3) if ε_A, ε_c, ε_d are normally distributed. Optimal study effort is

$$L_e^* = \arg\max_{L_e}\left\{\ln(M - T - AL_e^2) + \frac{1}{1+\delta}\left(\ln(A/4) + 2\left(\ln\left(L_e\right) + \mu_A + \mu_c + P_d\left(\mu_d - \mu_c\right) + \xi\right)\right)\right\}$$

where $\xi \equiv \sigma_c E\left[\varepsilon_c | \sigma_c\varepsilon_c - \sigma_d\varepsilon_d > \mu_d - \mu_c\right] + \sigma_d E\left[\varepsilon_d | \sigma_c\varepsilon_c - \sigma_d\varepsilon_d < \mu_d - \mu_c\right]$ can be written out explicitly in terms of normal densities as in Section 2.3. It is irrelevant to optimal effort, which as the positive solution of the first-order condition is

$$L_e^* = \sqrt{\frac{1}{2+\delta}\frac{(M-T)}{A}},$$

and in this simple log-linear specification of utility and labor income uncertainty does not depend on the moments of future random realizations. Inserting it in (A.3) yields the summary expectations and news expressions used in the main text's derivations.

B.3 Consumption choice

To focus on consumption and savings let values depend only on financial resources and wages,

$$v_j(A', M', \varepsilon_j) = \tilde{v}_j((M - C_j - T)R + w_j)$$

for $j = o, c, d$. Write $w_j = \mu_j + \varepsilon_j$, with $E\left[\varepsilon_j\right] = 0$ and $\varepsilon_d \equiv 0$ for notational simplicity, so dropout is the optimal ex post choice if $\tilde{v}_d((M - C - T)R + \mu_d) \geq \tilde{v}_c((M - C - T)R + \mu_c + \varepsilon_c)$ where C is consumption while enrolled. The realization of ε_c that triggers dropout is implicitly defined by

$$\tilde{v}_d((M - C - T)R + \mu_d) = \tilde{v}_c((M - C - T)R + \mu_c + \bar{\varepsilon}) \tag{B.8}$$

and depends on C if the value functions have different slopes at the critical point.

Denoting with $f_{\varepsilon_c}(\cdot)$ the density of ε_c,

$$C_e^* = \arg\max_C u_e(C) + \frac{1}{1+\delta}\left(\int_{-\infty}^{\bar{\varepsilon}(C)} \tilde{v}_c((M - C - T)R + \mu_d) f_{\varepsilon_c}(x)dx + \int_{\bar{\varepsilon}(C)}^{\infty} \tilde{v}_d((M - C - T)R + \mu_c + x) f_{\varepsilon_c}(x)dx\right)$$

$$\text{s.t.} \quad M - C - T \geq \bar{M}$$

is unique when utility is concave, generally depends on the expectations and variances (and other moments) of future wages, and can be inserted in (A.3).

When binding, the borrowing limit $\bar{M}$ implies $C_e = M - \bar{M} - T$, and the value of enrollment is lower than when borrowing is unconstrained. Binding borrowing limits similarly lower the value of continuation choices that imply a constrained consumption path in subsequent periods.

An explicit expression for C_e^* is available in some special cases. Suppose $\tilde{v}_d(\cdot)$ and $\tilde{v}_c(\cdot)$ have the same monotone increasing functional form $\tilde{v}_d(z) = \tilde{v}_c(z) = \tilde{v}(z)\ \forall z$. Then, dropout is optimal when $\varepsilon_c \leq \mu_d - \mu_c$ and consumption while enrolled, like study effort above, does not influence the dropout probability. Suppose ε_c is uniformly distributed with constant density $\gamma = \left(\sigma_\varepsilon 2\sqrt{3}\right)^{-1}$ for $-\sigma_\varepsilon\sqrt{3} < \varepsilon < \sigma_\varepsilon\sqrt{3}$, zero otherwise. Integrating the right-hand side of the first-order condition

$$u_e'(C)\frac{1+\delta}{R} = \int_{-\sigma_\varepsilon\sqrt{3}}^{\mu_d-\mu_c} \tilde{v}'((M-C-T)R+\mu_d)\gamma dx + \int_{\mu_d-\mu_c}^{\sigma_\varepsilon\sqrt{3}} \tilde{v}'((M-C-T)R+\mu_c+x)\gamma dx$$

yields

$$u_e'(C)\frac{1+\delta}{R\gamma} = v'((M-C-T)R+\mu_d)\left(\mu_d-\mu_c+\sqrt{3}\sigma_\varepsilon\right)+v((M-C-T)R+\mu_c+\sigma_\varepsilon\sqrt{3})-v((M-C-T)R+\mu_d).$$

If the functional forms of $u_e(\cdot)$ and $v(\cdot)$ are quadratic, with parameters ensuring that marginal utility is positive, C_e^* is the positive root of a quadratic equation. If the limits of integration depend on consumption while enrolled, as they do if functional forms differ on the two sides of (B.8), the difference of (quadratic) values at the critical $\tilde{\varepsilon}$ also appears in the first-order condition.

B.4 Many choices and shocks

The distribution of the value news denoted ϵ in the main text does not have a simple form when the linear budget constraint interacts with nonlinear preferences. The main text's derivations are valid if it is not substantially and predictably affected by parameter changes, or if a log-linear approximation is acceptably accurate. More flexible and realistic specifications of utility and of possible choices obviously complicate the solution, but allowing for multiple shocks and many choices make an approximately normal distribution plausible for news about the value of education.

Appendix C

With $dH\left(z\right)=dG\left(z\right)=\varphi(z)dz=\exp(-z^2/2)/\sqrt{2\pi}dz$ and using

$$\int_{z=x+y_d-y_c}^{\infty} z\exp(-\left(z/\sigma_c\right)^2/2)dz=\left(\sigma_c\right)^2\exp\left(-\frac{1}{2}\left(\frac{z+y_d-y_c}{\sigma_c}\right)^2\right)$$

to evaluate the inner integral in (10), that truncated convolution expectation reads

$$\int\limits_{-\infty}^{\infty}\sigma_c\left(\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{z+y_d-y_c}{\sigma_c}\right)^2}\right)\frac{1}{\sigma_d\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x}{\sigma_d}\right)^2}dz=\frac{\sigma_c}{\sigma_d}\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}e^{-\frac{1}{2}\left(\frac{\epsilon_d+y_d-y_c}{\sigma_c}\right)^2}e^{-\frac{1}{2}\left(\frac{\epsilon_d}{\sigma_d}\right)^2}dz$$

or, changing variables to $x=z/\sigma_d$,

$$\begin{aligned}&\frac{\sigma_c}{\sigma_d}\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-\frac{1}{2}\left(\left(x\frac{\sigma_d}{\sigma_c}+\frac{y_d-y_c}{\sigma_c}\right)^2+(x)^2\right)}\sigma_d dx=\\ &=\left|\frac{\sigma_c}{\sqrt{2\pi}}\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}}e^{-\frac{1}{2}\left(\frac{y_d-y_c}{\sqrt{\sigma_c^2+\sigma_d^2}}\right)^2}\operatorname{erf}\left(\frac{1}{\sqrt{2}}\frac{x\sigma_c^2+\frac{y_d-y_c}{\sigma_c}\sigma_c\sigma_d+x\sigma_d^2}{\sqrt{\sigma_c^2\left(\sigma_c^2+\sigma_d^2\right)}}\right)\right|_{x=-\infty}^{\infty}\\ &=\frac{\sigma_c}{\sqrt{2\pi}}\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}}e^{-\frac{1}{2}\left(\frac{y_d-y_c}{\sqrt{\sigma_c^2+\sigma_d^2}}\right)^2}\end{aligned}$$

where $\operatorname{erf}(z)=\frac{2}{\sqrt{\pi}}\int_o^z e^{-t^2}dt$ is the error function, and the last step uses $\operatorname{erf}\left(\infty\right)-\operatorname{erf}\left(-\infty\right)=2$. Thus, in the Gaussian case (10) can be written

$$E\left[\epsilon_c|\epsilon_d-\epsilon_c<y_c-y_d\right]\left(1-P_d\right)=\sigma_c\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}}\varphi\left(\frac{y_d-y_c}{\sqrt{\sigma_c^2+\sigma_d^2}}\right).$$

Symmetric derivations for (11) yield

$$E\left[\epsilon_d|\epsilon_c-\epsilon_d<y_d-y_c\right]P_d=\sigma_d\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}}\varphi\left(\frac{y_d-y_c}{\sqrt{\sigma_c^2+\sigma_d^2}}\right).$$

Inserting these and (2) in (4), and using

$$\sigma_c\sqrt{\frac{\sigma_c^2}{\sigma_c^2+\sigma_d^2}}+\sigma_d\sqrt{\frac{\sigma_d^2}{\sigma_c^2+\sigma_d^2}}=\sqrt{\sigma_c^2+\sigma_d^2}\equiv\sigma$$

to simplify the resulting expression, yields (12).

Appendix D

Suppose the payoffs y_o, y_c, y_d in the numerator of the scaled differences defined in (14) are normally distributed in the population and independent of each other, and the spread σ of news is for simplicity constant across individuals. (Underlying sources of variation will generally influence both expectations and the extent of uncertainty around them in a given population, but numerical experiments where σ is drawn from a χ^2_1 distribution yield qualitatively similar illustrations.)

The means $\tilde{\mu}_d$ and $\tilde{\mu}_o$, standard deviations $\tilde{\sigma}_d$ and $\tilde{\sigma}_o$, and correlation ρ of $\tilde{y}_d$ and $\tilde{y}_o$ are related to the means and variances of the cross-sectional distribution of individual payoffs by

$$\tilde{\mu}_d = \text{mean}(y_d) - \text{mean}(y_c),\ \tilde{\mu}_o = \text{mean}(y_o) - \text{mean}(y_c),$$
$$\tilde{\sigma}_d = \sqrt{\text{var}(y_c)+\text{var}(y_d)},\ \tilde{\sigma}_o = \sqrt{\text{var}(y_c)+\text{var}(y_o)},\ \rho = \frac{\text{var}(y_c)}{\tilde{\sigma}_d\tilde{\sigma}_o}.$$

The correlation ρ is generally positive, as y_c appears in both scaled differences with the same sign, and close to unity if most cross-sectional variation is driven by y_c.

In the top panel the figure plots density contour lines. How many individuals enroll at each $\tilde{y}_d$ depends on the distribution of $\tilde{y}_o$ conditional on $\tilde{y}_d$, which is normal with mean $\tilde{\mu}_o + \rho\left(\tilde{y}_d - \tilde{\mu}_d\right)\tilde{\sigma}_o/\tilde{\sigma}_d$ and variance $(1-\rho^2)\tilde{\sigma}_o^2$. Hence,

$$\text{prob}\left(\tilde{y}_o < \varphi\left(\tilde{y}_d\right) + \tilde{y}_d\Phi\left(\tilde{y}_d\right)|\tilde{y}_d\right) = \Phi\left(\frac{\varphi\left(\tilde{y}_d\right) + \tilde{y}_d\Phi\left(\tilde{y}_d\right) - \left(\tilde{\mu}_o + \rho\left(\tilde{y}_d - \tilde{\mu}_d\right)\tilde{\sigma}_o/\tilde{\sigma}_d\right)}{\sqrt{(1-\rho^2)\tilde{\sigma}_o^2}}\right).$$

Whether this expression is increasing or decreasing in $\tilde{y}_d$ depends on whether the conditional density of $\tilde{y}_o$ grows faster or slower than the enrollment trigger.

Integration over the marginal distribution of $\tilde{y}_d$ yields the population enrollment rate

$$\int_{-\infty}^{\infty}\Phi\left(\frac{\varphi\left(\tilde{y}_d\right) + \tilde{y}_d\Phi\left(\tilde{y}_d\right) - \left(\tilde{\mu}_o + \rho\left(\tilde{y}_d - \tilde{\mu}_d\right)\tilde{\sigma}_o/\tilde{\sigma}_d\right)}{\sqrt{(1-\rho^2)\tilde{\sigma}_o^2}}\right)\varphi\left(\frac{\tilde{y}_d - \tilde{\mu}_d}{\tilde{\sigma}_d}\right)d\tilde{y}_d$$

and the aggregate dropout rate of the enrolled

$$\int_{-\infty}^{\infty}\Phi\left(\tilde{y}_d\right)\Phi\left(\frac{\varphi\left(\tilde{y}_d\right) + \tilde{y}_d\Phi\left(\tilde{y}_d\right) - \left(\tilde{\mu}_o + \rho\left(\tilde{y}_d - \tilde{\mu}_d\right)\tilde{\sigma}_o/\tilde{\sigma}_d\right)}{\sqrt{(1-\rho^2)\tilde{\sigma}_o^2}}\right)\varphi\left(\frac{\tilde{y}_d - \tilde{\mu}_d}{\tilde{\sigma}_d}\right)d\tilde{y}_d.$$

References

Athreya, Kartik and Eberly, Janice C. (2021) "Risk, the College Premium, and Aggregate Human Capital Investment" American Economic Journal: Macroeconomics, 13:2, 168-213

Bound, John, Michael F. Lovenheim, and Sarah Turner (2010) "Why Have College Completion Rates Declined? An Analysis of Changing Student Preparation and Collegiate Resources" American Economic Journal: Applied Economics 2:3, 129-57

Cameron, Stephen V., and James J. Heckman (2001) "The Dynamics of Educational Attainment for Black, Hispanic, and White Males" Journal of Political Economy 109:3, 455-499

Chatterjee, Satyajit and Felicia Ionescu (2012) "Insuring Student Loans against the Financial Risk of Failing to Complete College" Quantitative Economics 3, 393–420

Comay, Yochanan, Arie Melnik and Moshe A. Pollatschek (1976) "Dropout Risks, Option Values, and Returns to Investment in Schooling" The Canadian Journal of Economics / Revue canadienne d'Economique 9:1, 45-56

Contini, Dalit, Federica Cugnata, and Andrea Scagni (2018) "Social Selection in Higher Education: Enrollment, dropout and timely degree attainment in Italy" Higher Education 75:5, 785-808

Dunning, David and Justin Kruger (1999) "Unskilled and Unaware of It: How difficulties in recognizing one's own incompetence lead to inflated self-assessments" Journal of Personality and Social Psychology 77(6), 1121-34

Ghignoni, Emanuela (2016) "Family Background and University Dropouts During the Crisis: the case of Italy" Higher Education 71(4), 1-25

Hendricks, Lutz and Oksana Leukhina (2018) "The Return To College: Selection And Dropout Risk" International Economic Review 59:3, 1077-1102And

Manski, Charles F. (1989) "Schooling as Experimentation: A Reappraisal of the Postsecondary Dropout Phenomenon" Economics of Education Review 8(4), 305–12

Ozdagli, Ali, and Nicholas Trachter (2014) "The Dropout Option in a Simple Model of College Education," Economic Quarterly, Federal Reserve Bank of Richmond, issue 4Q, pages 279-295.

Paul, Marie (2015) "Many Dropouts? Never Mind! Employment Prospects of Dropouts from Training Programs" Annals of Economics and Statistics 119/120, 235-267

Stange, Kevin M. (2012) "An Empirical Investigation of the Option Value of College Enrollment" American Economic Journal: Applied Economics 4:1, 49-84

Stinebrickner, Ralph and Todd Stinebrickner (2008) "The Effect of Credit Constraints on the College Drop-Out Decision: A Direct Approach Using a New Panel Study" American Economic Review 98:5, 2163-2184

Stinebrickner, Todd and Ralph Stinebrickner (2012) "Learning about Academic Ability and the College Dropout Decision" Journal of Labor Economics 30:4 707-748

Stinebrickner, Todd and Ralph Stinebrickner (2014) "Academic Performance and College Dropout: Using Longitudinal Expectations Data to Estimate a Learning Model" Journal of Labor Economics 32:3 601-44

Vossensteyn, Hans, Bjørn Stensaker, Andrea Kottmann, Elisabeth Hovdhaugen, Ben Jongbloed, Sabine Wollscheid, Frans Kaiser, and Leon Cremonini (2015) "Dropout and Completion in Higher Education in Europe" Brussels: European Commission.